

QUANTUM DUALITY PRINCIPLE FOR QUANTUM CONTINUOUS KAC–MOODY ALGEBRAS

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ABSTRACT. For the quantized universal enveloping algebra $U_{\hbar}(\mathfrak{g}_X)$ associated to a continuous Kac-Moody algebra \mathfrak{g}_X as in [ApS20], we prove that a suitable formulation of the *Quantum Duality Principle* holds true, both in a “formal” version — i.e., applying to the original definition of $U_{\hbar}(\mathfrak{g}_X)$ as a formal QUEA over $\mathbb{k}[[\hbar]]$ — and in a “polynomial” one — i.e., for a suitable polynomial form of $U_{\hbar}(\mathfrak{g}_X)$ over $\mathbb{k}[q, q^{-1}]$. In both cases, the QDP states that a suitable subalgebra of the given quantization of the Lie bialgebra \mathfrak{g}_X is in fact a suitable quantization (in formal or in polynomial sense) of a connected Poisson group G_X^* dual to \mathfrak{g}_X .

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