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*“Twisted deformations vs. cocycle deformations
for quantum groups”*

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ABSTRACT

In this paper we study two deformation procedures for quantum groups: deformations by twists, that we call “*comultiplication twisting*”, as they modify the coalgebra structure, while keeping the algebra one — and deformations by 2–cocycle, that we call “*multiplication twisting*”, as they deform the algebra structure, but save the coalgebra one.

We deal with quantum universal enveloping algebras, in short QUEA’s, for which we accordingly consider those arising from twisted deformations (in short TwQUEA’s) and those arising from 2–cocycle deformations, usually called multiparameter QUEA’s (in short MpQUEA’s). Up to technicalities, we show that the two deformation methods are equivalent, in that they eventually provide isomorphic outputs, which are deformations (of either kinds) of the “canonical”, well-known one-parameter QUEA by Jimbo and Lusztig. It follows that the two notions of TwQUEA’s and of MpQUEA’s — which, in Hopf algebra theoretical terms are naturally dual to each other — actually coincide; thus, that there exists in fact only one type of “pluriparametric deformation” for QUEA’s. In particular, the link between the realization of any such QUEA as a MpQUEA and that as a TwQUEA is just a (very simple, and rather explicit) change of presentation.

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