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*“A new equivalence between
super Harish-Chandra pairs and Lie supergroups”*

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ABSTRACT

It is known that there exists a natural functor Φ from Lie supergroups to super Harish-Chandra pairs. A functor going backwards, that associates a Lie supergroup with each super Harish-Chandra pair, yielding an equivalence of categories, was found by Koszul [19], and later generalized by several authors.

In this paper, we provide two new backwards equivalences, i.e. two different functors Ψ° and Ψ^e that construct a Lie supergroup (thought of as a special group-valued functor) out of a given super Harish-Chandra pair, so that both Ψ° and Ψ^e are quasi-inverse to the functor Φ .

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