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"Duality features of left Hopf algebroids"

ABSTRACT

We explore special features of the pair (U^*, U_*) formed by the right and left dual over a (left) bialgebroid U in case the bialgebroid is, in particular, a left Hopf algebroid. It turns out that there exists a bialgebroid morphism S^* from one dual to another that extends the construction of the antipode on the dual of a Hopf algebra, and which is an isomorphism if U is both a left and right Hopf algebroid. This structure is derived from Phùng's categorical equivalence between left and right comodules over U without the need of a (Hopf algebroid) antipode, a result which we review and extend. In the applications, we illustrate the difference between this construction and those involving antipodes and also deal with dualising modules and their quantisations.

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