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"Global splittings and super Harish-Chandra pairs for affine supergroups"

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ABSTRACT

This paper dwells upon two aspects of affine supergroup theory, investigating the links among them.

First, I discuss the "splitting" properties of affine supergroups, i.e. special kinds of factorizations they may admit — either globally, or point-wise. Almost everything should be more or less known, but seems to be not as clear in literature (to the author's knowledge) as it ought to.

Second, I present a new contribution to the study of affine supergroups by means of super Harish-Chandra pairs (a method already introduced by Koszul, and later extended by other authors). Namely, I provide a new functorial construction Ψ which, with each super Harish-Chandra pair, associates an affine supergroup that is always *globally strongly split* (in short, *gs-split*) — thus setting a link with the first part of the paper. One knows that there exists a natural functor Φ from affine supergroups to super Harish-Chandra pairs: then I show that the new functor Ψ — which goes the other way round — is indeed a quasi-inverse to Φ, provided we restrict our attention to the subcategory of affine supergroups that are gs-split. Therefore, (the restrictions of) Φ and Ψ are equivalences between the categories of gs-split affine supergroups and of super Harish-Chandra pairs. Such a result was known in other contexts, such as the smooth differential or the complex analytic one, via different approaches (see [16], [19], [7]): nevertheless, the novelty in the present paper lies in that I construct a *different* functor Ψ and thus extend the result to a much larger setup, with a totally different, more geometrical method. In fact, this method (very concrete, indeed) is universal and characteristic-free: I present it here for the algebro-geometric setting, but actually it can be easily adapted to the frameworks of differential or complex-analytic supergeometry.

The case of *linear* supergroups is treated also as an intermediate, inspiring step.

Some examples, applications and further generalizations are presented at the end of the paper.

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