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"A global quantum duality principle for subgroups and homogeneous spaces"

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ABSTRACT

For a complex or real algebraic group G, with $\mathfrak{g} := \mathrm{Lie}(G)$, quantizations of *global* type are suitable Hopf algebras $F_q[G]$ or $U_q(\mathfrak{g})$ over $\mathbb{C}\big[q, q^{-1}\big]$. Any such quantization yields a structure of Poisson group on G, and one of Lie bialgebra on $\mathfrak{g}$: correspondingly, one has dual Poisson groups G^* and a dual Lie bialgebra $\mathfrak{g}^*$. In this context, we introduce suitable notions of *quantum subgroup* and, correspondingly, of *quantum homogeneous space*, in three versions: *weak*, *proper* and *strict* (also called *flat* in the literature). The last two notions only apply to those subgroups which are coisotropic, and those homogeneous spaces which are Poisson quotients; the first one instead has no restrictions whatsoever.

The global quantum duality principle (GQDP), as developed in [12], associates with any global quantization of G, or of $\mathfrak{g}$, a global quantization of $\mathfrak{g}^*$, or of G^*. In this paper we present a similar GQDP for quantum subgroups or quantum homogeneous spaces. Roughly speaking, this associates with every quantum subgroup, resp. quantum homogeneous space, of G, a quantum homogeneous space, resp. a quantum subgroup, of G^*. The construction is tailored after four parallel paths — according to the different ways one has to algebraically describe a subgroup or a homogeneous space — and is "functorial", in a natural sense.

Remarkably enough, the output of the constructions are always quantizations of *proper* type. More precisely, the output is related to the input as follows: the former is the *coisotropic dual* of the coisotropic interior of the latter — a fact that extends the occurrence of Poisson duality in the original GQDP for quantum groups. Finally, when the input is a strict quantization then the output is strict as well — so the special rôle of strict quantizations is respected.

We end the paper with some explicit examples of application of our recipes.

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