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"Duality functors for quantum groupoids"

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ABSTRACT

We present a formal algebraic language to deal with quantum deformations of Lie-Rinehart algebras — or Lie algebroids, in a geometrical setting. In particular, extending the ice-breaking ideas introduced by Xu in [34], we provide suitable notions of "quantum groupoids". For these objects, we detail somewhat in depth the formalism of linear duality; this yields several fundamental antiequivalences among (the categories of) the two basic kinds of "quantum groupoids". On the other hand, we develop a suitable version of a "quantum duality principle" for quantum groupoids, which extends the one for quantum groups — dealing with Hopf algebras — originally introduced by Drinfeld (cf. [8], §7) and later detailed in [12].

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