Rita FIORESI, Fabio GAVARINI

"Chevalley Supergroups"

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ABSTRACT

In the framework of algebraic supergeometry, we give a construction of the scheme-theoretic supergeometric analogue of Chevalley groups, namely affine algebraic supergroups associated to simple Lie superalgebras of classical type. This provides a unified approach to most of the algebraic supergroups considered so far in literature, and an effective method to construct new ones. As an intermediate step, we prove an existence theorem for Chevalley bases of simple classical Lie superalgebras and a PBW-like theorem for their associated Kostant superalgebras.

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