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“On the radical of Brauer algebras”

Mathematische Zeitschrift **260** (2008), 673–697 — `DOI: 10.1006/jabr.1996.7003`
— *the original publication is available at* `www.springerlink.com` —

ABSTRACT

The radical of the Brauer algebra $\mathcal{B}_f^{(x)}$ is known to be non-trivial when the parameter x is an integer subject to certain conditions (with respect to f). In these cases, we display a wide family of elements in the radical, which are explicitly described by means of the diagrams of the usual basis of $\mathcal{B}_f^{(x)}$. The proof is by direct approach for $x = 0$, and via classical Invariant Theory in the other cases, exploiting then the well-known representation of Brauer algebras as centralizer algebras of orthogonal or symplectic groups acting on tensor powers of their standard representation. This also gives a great part of the radical of the generic indecomposable $\mathcal{B}_f^{(x)}$–modules. We conjecture that this part is indeed the whole radical in the case of modules, and it is the whole part in a suitable step of the standard filtration in the case of the algebra. As an application, we find some more precise results for the module of pointed chord diagrams, and for the Temperley-Lieb algebra — realised inside $\mathcal{B}_f^{(1)}$ — acting on it.

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