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# *"$F_q[M_n]$, $F_q[GL_n]$ and $F_q[SL_n]$ as quantized hyperalgebras"*

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## ABSTRACT

Within the quantum function algebra $F_q[GL_n]$, we study the subset $\mathcal{F}_q[GL_n]$ — introduced in [Ga1] — of all elements of $F_q[GL_n]$ which are $\mathbb{Z}\big[q,q^{-1}\big]$–valued when paired with $\mathcal{U}_q(\mathfrak{gl}_n)$, the unrestricted $\mathbb{Z}\big[q,q^{-1}\big]$–integral form of $U_q(\mathfrak{gl}_n)$ introduced by De Concini, Kac and Procesi. In particular we obtain a presentation of it by generators and relations, and a PBW-like theorem. Moreover, we give a direct proof that $\mathcal{F}_q[GL_n]$ is a Hopf subalgebra of $F_q[GL_n]$, and that $\mathcal{F}_q[GL_n]\Big|_{q=1} \cong U_{\mathbb{Z}}\big({\mathfrak{gl}_n}^*\big)$, that is the Kostant-like $\mathbb{Z}$–form of $U\big({\mathfrak{gl}_n}^*\big)$ generated by divided powers of root vectors. We describe explicitly its specializations at roots of 1, say $\varepsilon$, and the associated quantum Frobenius (epi)morphism from $\mathcal{F}_\varepsilon[GL_n]$ to $\mathcal{F}_1[SL_n] \cong U_{\mathbb{Z}}\big({\mathfrak{gl}_n}^*\big)$, also introduced in [Ga1]. The same analysis is done for $\mathcal{F}_q[SL_n]$ and (as key step) for $\mathcal{F}_q[Mat_n]$.

DEDICATORY

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*This paper is dedicated to the memory of all victims of that war.*

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## References

[APW] H. H. Andersen, P. Polo, K. Wen, *Representations of quantum algebras*, Inventiones Mathematicæ **104** (1991), 1–59.

[BGY] K. A. Brown, K. R. Goodearl, M. Yakimov, *Poisson structures on affine spaces and flag varieties. I. Matrix affine Poisson space*, Advances in Mathematics **206** (2006), 567–629.

[CP] V. Chari, A. Pressley, *A guide to Quantum Groups*, Cambridge Univ. Press, Cambridge, 1994.

[DL] C. De Concini, V. Lyubashenko, *Quantum function algebra at roots of 1*, Advances in Mathematics **108** (1994), 205–262.

[DP] C. De Concini, C. Procesi, *Quantum groups*, D-modules, Representation Theory, and Quantum Groups (L. Boutet de Monvel, C. De Concini, C. Procesi, P. Schapira, M. Vergne, eds.), Lecture Notes in Mathematics **1565**, Springer-Verlag, Berlin–Heidelberg–New York, 1993, pp. 798–820.

[Ga1] F. Gavarini, *Quantization of Poisson groups*, Pacific Journal of Mathematics **186** (1998), 217–266.

[Ga2] ———, *Quantum function algebras as quantum enveloping algebras*, Communications in Algebra **26** (1998), 1795–1818.

[Ga3] ———, *Presentation by Borel subalgebras and Chevalley generators for quantum enveloping algebras*, Proceedings of the Edinburgh Mathematical Society **49** (2006), 291–308.

[Ga4] ———, *PBW theorems and Frobenius structures for quantum matrices*, electronic preprint posted at `http://arxiv.org/abs/math.QA/0610691` (2006), 9 pages.

[GR] F. Gavarini, Z. Rakić, *$F_q[Mat_2]$, $F_q[GL_2]$ and $F_q[SL_2]$ as quantized hyperalgebras*, electronic preprint posted at `http://arxiv.org/abs/math.QA/0411440` (2004), 40 pages.

[Hu] J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Graduate Texts in Mathematics **9**, Springer-Verlag, Berlin–Heidelberg–New York, 1972.

[Ji] M. Jimbo,, *A $q$–analogue of $U(gl(N+1))$, Hecke Algebras and the Yang Baxter equation*, Lett. Math. Phys. **10** (1985), 63–69.

[Lu] G. Lustzig, *Quantum groups at roots of 1*, Geom. Dedicata **35** (1990), 89–113.

[No] M. Noumi, *Macdonald's Symmetric Polynomials as Zonal Spherical Functions on Some Quantum Homogeneous Spaces*, Adv. Math. **123** (1996), 16–77.