F. Gavarini

"The global quantum duality principle"

Journal für die reine und angewandte Mathematik (Crelle's Journal) **612** (2007), 17–33.

ABSTRACT

Let R be an integral domain, let $\hbar \in R \setminus \{0\}$ be such that $\Bbbk := R \big/ \hbar R$ is a field, and let $\mathcal{HA}$ be the category of torsionless (or flat) Hopf algebras over R. We call $H \in \mathcal{HA}$ a *"quantized function algebra" (=QFA)*, resp. *"quantized restricted universal enveloping algebras" (=QrUEA)*, at $\hbar$ if — roughly speaking — $H \big/ \hbar H$ is the function algebra of a connected Poisson group, resp. the (restricted, if $R \big/ \hbar R$ has positive characteristic) universal enveloping algebra of a (restricted) Lie bialgebra. Extending a result of Drinfeld, we establish an "inner" Galois' correspondence on $\mathcal{HA}$, via two endofunctors, $(\ )^\vee$ and $(\ )'$, of $\mathcal{HA}$ such that $H^\vee$ is a QrUEA and H' is a QFA (for all $H \in \mathcal{HA}$). In addition:

(a) the image of $(\ )^\vee$, resp. of $(\ )'$, is the full subcategory of all QrUEAs, resp. QFAs;

(b) if $p := \mathit{Char}(\Bbbk) = 0$, the restrictions $(\ )^\vee\big|_{\text{QFAs}}$ and $(\ )'\big|_{\text{QrUEAs}}$ yield equivalences inverse to each other;

(c) if $p = 0$, starting from a QFA over a Poisson group G, resp. from a QrUEA over a Lie bialgebra $\mathfrak{g}$, the functor $(\ )^\vee$, resp. $(\ )'$, gives a QrUEA, resp. a QFA, over the dual Lie bialgebra, resp. the dual Poisson group.

Several, far-reaching applications are developed in detail in [Ga2–4].

— — — — —

References

[Bo] N. Bourbaki, *Commutative Algebra*, Springer & Verlag, New York-Heidelberg-Berlin-Tokyo, 1989.

[CP] V. Chari, A. Pressley, *A guide to Quantum Groups*, Cambridge University Press, Cambridge, 1994.

[Dr] V. G. Drinfeld, *Quantum groups*, Proc. Intern. Cong. of Math. (Berkeley, 1986), 1987, pp. 798–820.

[EH] B. Enriquez, G. Halbout, *An $\hbar$–adic valuation property of universal R–matrices*, J. Algebra **261** (2003), 434–447.

[EK] P. Etingof, D. Kazhdan, *Quantization of Lie bialgebras, I*, Selecta Math. (N.S.) **2** (1996), 1–41.

[Ga1] F. Gavarini, *The quantum duality principle*, Annales de l'Institut Fourier **52** (2002), 809–834.

[Ga2] ———, *The global quantum duality principle: theory, examples, and applications*, electronic preprint `http://arxiv.org/abs/math.QA/0303019` (2003).

[Ga3] ———, *The Crystal Duality Principle: from Hopf Algebras to Geometrical Symmetries*, Journal of Algebra **285** (2005), 399–437.

[Ga4] ———, *Poisson geometrical symmetries associated to non-commutative formal diffeomorphisms*, Communications in Mathematical Physics **253** (2005), 121–155.

[KT] C. Kassel, V. Turaev, *Biquantization of Lie bialgebras*, Pac. Jour. Math. **195** (2000), 297–369.

[Mo] S. Montgomery, *Hopf Algebras and Their Actions on Rings*, CBMS Regional Conference Series in Mathematics **82**, American Mathematical Society, Providence, RI, 1993.