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"Presentation by Borel subalgebras and Chevalley generators for quantum enveloping algebras"

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ABSTRACT

We provide an alternative approach to the Faddeev-Reshetikhin-Takhtajan presentation of the quantum group $U_q(\mathfrak{g})$, with L–operators as generators and relations ruled by an R–matrix. We look at $U_q(\mathfrak{g})$ as being generated by the quantum Borel subalgebras $U_q(\mathfrak{b}_+)$ and $U_q(\mathfrak{b}_-)$, and use the standard presentation of the latters as quantum function algebras. When $\mathfrak{g} = \mathfrak{gl}_n$ these Borel quantum function algebras are generated by the entries of a triangular q–matrix, thus eventually $U_q(\mathfrak{gl}_n)$ is generated by the entries of an upper triangular and a lower triangular q–matrix, which share the same diagonal. The same elements generate over $\Bbbk\big[q, q^{-1}\big]$ the unrestricted $\Bbbk\big[q, q^{-1}\big]$–integer form of $U_q(\mathfrak{gl}_n)$ of De Concini and Procesi, which we present explicitly, together with a neat description of the associated quantum Frobenius morphisms at roots of 1. All this holds, *mutatis mutandis*, for $\mathfrak{g} = \mathfrak{sl}_n$ too.

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