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"A quantum duality principle for coisotropic subgroups and Poisson quotients"

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ABSTRACT

We develop a quantum duality principle for coisotropic subgroups of a (formal) Poisson group and its dual: namely, starting from a quantum coisotropic subgroup (for a quantization of a given Poisson group) we provide functorial recipes to produce quantizations of the dual coisotropic subgroup (in the dual formal Poisson group). By the natural link between subgroups and homogeneous spaces, we argue a quantum duality principle for Poisson homogeneous spaces which are Poisson quotients, i.e. have at least one zero-dimensional symplectic leaf. As an application, we provide an explicit quantization of the homogeneous SL_n^*–space of Stokes matrices, with the Poisson structure given by Dubrovin and Ugaglia.

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