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"The Crystal Duality Principle: from Hopf Algebras to Geometrical Symmetries"

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ABSTRACT

We give functorial recipes to get, out of any Hopf algebra over a field, two pairs of Hopf algebras with some geometrical content. If the ground field has characteristic zero, the first pair is made of a function algebra $F[G_+]$ over a connected Poisson group and a universal enveloping algebra $U(\mathfrak{g}_-)$ over a Lie bialgebra $\mathfrak{g}_-$. In addition, the Poisson group as a variety is an affine space, and the Lie bialgebra as a Lie algebra is graded. Forgetting these last details, the second pair is of the same type, namely $\big(F[K_+], U(\mathfrak{k}_-)\big)$ for some Poisson group K_+ and some Lie bialgebra $\mathfrak{k}_-$. When the Hopf algebra H we start from is already of geometric type, the result involves Poisson duality. The first Lie bialgebra associated to $H = F[G]$ is $\mathfrak{g}^*$ (with $\mathfrak{g} := \mathit{Lie}(G)$), and the first Poisson group associated to $H = U(\mathfrak{g})$ is of type G^*, i.e. it has $\mathfrak{g}$ as cotangent Lie bialgebra. If the ground field has positive characteristic, the same recipes give similar results, but the Poisson groups obtained have dimension 0 and height 1, and *restricted* universal enveloping algebras are obtained. We show how these geometrical Hopf algebras are linked to the initial one via 1-parameter deformations, and explain how these results follow from quantum group theory. Finally, we examine in detail the case of group algebras.

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References

[Ab] N. Abe, *Hopf algebras*, Cambridge Tracts in Mathematics **74**, Cambridge University Press, Cambridge, 1980.

[CP] V. Chari, A. Pressley, *A guide to Quantum Groups*, Cambridge University Press, Cambridge, 1994.

[DG] M. Demazure, P. Gabriel, *Groupes Algébriques I*, North Holland, Amsterdam, 1970.

[Fo1] L. Foissy, *Les algèbres de Hopf des arbres enracinés décorés, I*, Bull. Sci. Math. **126** (2002), 193–239.

[Fo2] ———, *Les algèbres de Hopf des arbres enracinés décorés, II*, Bull. Sci. Math. **126** (2002), 249–288.

[Ga1] F. Gavarini, *The global quantum duality principle: theory, examples, and applications*, preprint http://arxiv.org/abs/math.QA/0303019 (2003).

[Ga2] ———, *The global quantum duality principle*, to appear (2006).

[Ga3] ———, *Poisson geometrical symmetries associated to non-commutative formal diffeomorphisms*, Comm. Math. Phys. **253** (2005), 121–155.

[HB] B. Huppert, N. Blackburn, *Finite Groups. II*, Grundlehren der Mathematischen Wissenschaften **243**, Springer & Verlag, Berlin – New York, 1982.

[KT] C. Kassel, V. Turaev, *Biquantization of Lie bialgebras*, Pac. Jour. Math. **195** (2000), 297–369.

[Mo] S. Montgomery, *Hopf Algebras and Their Actions on Rings*, CBMS Regional Conference Series in Mathematics **82**, American Mathematical Society, Providence, RI, 1993.

[Pa] D. S. Passman, *The Algebraic Structure of Group Rings*, Pure and Applied Mathematics, J. Wiley & Sons, New York, 1977.

[Sw] M. E. Sweedler, *Hopf Algebras*, Mathematics Lecture Note Series, W. A. Benjamin, Inc., New York, 1969.

[Tu1] V. G. Turaev, *Algebras of loops on surfaces, algebras of knots, and quantization,* in: C. N. Yang, M. L. Ge (eds.)*, Braid Groups, Knot Theory, Statistical Mechanics*, Advanced Series in Math. Phys. **9**, World Scientific, Singapore, 1989, 59–95.

[Tu2] ———, *Skein quantization of Poisson algebras of loops on surfaces*, Ann. Scient. Ec. Norm. Sup. 4e série **24** (1991), 635–704.