F. Gavarini, "Poisson geometrical symmetries associated to non-commutative formal diffeomorphisms"

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ABSTRACT

Let \mathcal{G}^{dif} be the group of all formal power series starting with x with coefficients in a field k of zero characteristic (with the composition product), and let $F[\mathcal{G}^{\text{dif}}]$ be its function algebra. In [BF] a non-commutative, non-cocommutative graded Hopf algebra \mathcal{H}^{dif} was introduced via a direct process of "disabelianisation" of $F[\mathcal{G}^{\text{dif}}]$, taking the like presentation of the latter as an algebra but dropping the commutativity constraint. In this paper we apply a general method to provide four one-parameters deformations of \mathcal{H}^{dif} , which are quantum groups whose semiclassical limits are Poisson geometrical symmetries such as Poisson groups or Lie bialgebras, namely two quantum function algebras and two quantum universal enveloping algebras. In particular the two Poisson groups are extensions of \mathcal{G}^{dif} , isomorphic as proalgebraic Poisson varieties but not as proalgebraic groups.

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