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"The crystal duality principle: from general symmetries to geometrical symmetries"

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ABSTRACT

We give functorial recipes to get, out of any Hopf algebra over a field, two pairs of Hopf algebras bearing some geometrical content. If the ground field has zero characteristic, the first pair is made of a function algebra $F[G_+]$ over a connected Poisson group and a universal enveloping algebra $U(\mathfrak{g}_-)$ over a Lie bialgebra $\mathfrak{g}_-$: in addition, the Poisson group as a variety is an affine space, and the Lie bialgebra as a Lie algebra is graded; apart for these last details, the second pair is of the same type, namely $\big(F[G_-], U(\mathfrak{g}_+)\big)$ for some Poisson group G_- and some Lie bialgebra $\mathfrak{g}_+$. When the Hopf algebra H we start from is already of geometric type the result involves Poisson duality: the first Lie bialgebra associated to $H = F[G]$ is $\mathfrak{g}^*$ (with $\mathfrak{g} := \mathit{Lie}(G)$), and the first Poisson group associated to $H = U(\mathfrak{g})$ is of type G^*, i.e. it has $\mathfrak{g}$ as cotangent Lie bialgebra. If the ground field has positive characteristic, then the same recipes give similar results, but for the fact that the Poisson groups obtained have dimension 0 and height 1, and *restricted* universal enveloping algebras are obtained. We show how all these "geometrical" Hopf algebras are linked to the original one via 1-parameter deformations, and explain how these results follow from quantum group theory.

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