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"A PBW basis for Lusztig's form of untwisted affine quantum groups"

INTRODUCTION

"Questa forma è duale
di un'altra già nota
che ha un suo teorem PBW.
Ed è subito base"

N. Barbecue, "Scholia"

Let $\hat{\mathfrak{g}}$ be an untwisted affine Kac-Moody algebra, and let $U_q(\hat{\mathfrak{g}})$ be the associated quantum enveloping algebra. In [Be1], [Be2], quantum root vectors are defined, and a basis of Poincaré-Birkhoff-Witt type for $U_q(\hat{\mathfrak{g}})$ is constructed, made of ordered monomials in the quantum root vectors.

Now let $\mathfrak{U}_q(\hat{\mathfrak{g}})$ be the Lusztig's integer form of $U_q(\hat{\mathfrak{g}})$, generated over $\mathbb{Z}\left[q, q^{-1}\right]$ by q–divided powers $E_i^{(n)}$, $F_i^{(m)}$; for technical reasons, we shall use a larger ground ring R. In this paper we find a PBW basis of $\mathfrak{U}_q(\hat{\mathfrak{g}})$ as an R–module, made of ordered products of q–divided powers of (suitable renormalizations of) quantum root vectors.

As a first step we reduce the problem to finding a basis for $\mathfrak{U}_q^+$, the positive part of $\mathfrak{U}_q(\hat{\mathfrak{g}})$. Second, we exploit the duality among PBW basis in U_q^+ and in U_q^- — proved in [Da2] — to get from there our key result, namely finding a basis of $\mathfrak{U}_q^+$.

Such an approach is entirely different from the classical ones, to be found in [Ga] and [Mi]. On the other hand, the comparison with the classical setting is quite interesting: this is sketched in the last section, where also a second PBW theorem is proved and some further conjectures are presented.

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