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"A Brauer algebra theoretic proof of Littlewood's restriction rules"

ABSTRACT

Let U be a complex vector space endowed with an orthogonal or symplectic form, and let G be the subgroup of $GL(U)$ of all the symmetries of this form (resp. $O(U)$ or $Sp(U)$); if M is an irreducible $GL(U)$–module, the Littlewood's restriction rule describes the G–module $M\Big|_{G}^{GL(U)}$. In this paper we give a new representation-theoretic proof of this formula: realizing M in a tensor power $U^{\otimes f}$ and using Schur's duality we reduce to the problem of describing the restriction to an irreducible S_f–module of an irreducible module for the centralizer algebra of the action of G on $U^{\otimes f}$; the latter is a quotient of the Brauer algebra, and we know the kernel of the natural epimorphism, whence we deduce the Littlewood's restriction rule.

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