SCIENTIFIC INTERESTS

of Fabio Gavarini

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Research main guidelines

My scientific activity till now has been centered around representation theory, Lie theories and their quantization: Lie algebras, Lie groups, algebraic groups and their representations, invariant theory, homogeneous spaces, quantum theories, etc. Within this very broad framework, my research work has focused upon three specific topics: algebraic groups and their representations, quantum groups, Hopf algebras and their generalizations. Hereafter I sketch a rough outline of my results about all this (*Remark:* the numbers in square brackets below refer to the subsequent list of publications).

Algebraic Groups, Representations and related topics: In the broad field of algebraic group theory and its representation-theoretical side, I mainly focused upon two main trends: classical invariant theory, and (algebraic) supergroup theory.

Classical Invariant Theory: In invariant theory for classical groups a central role is played by the so-called Schur-Brauer-Weyl duality. This strictly relates the irreducible representations of such a group to the irreducible representations of the algebra $A(m)$ which centralizes the action of the group itself on $V^{\otimes m}$, where V is the natural representation (over $\mathbf{C}$) of the group. For $GL(V)$ or $SL(V)$ the algebra $A(m)$ is a quotient of the group algebra of the symmetric group on m elements; when realizing the latter as set of graphs, and its product as a "graphical composition", one obtains a combinatorial description of $A(m)$. For the orthogonal or symplectic groups instead, $A(m)$ is a quotient of the so-called *Brauer algebra*, which has as basis a semigroup of graphs which extends the symmetric group, and still admits a suitable combinatorial description. For the last one hundred years this theory has been, and still is nowadays, of central importance for the representation theory of classical groups; it cyclically comes back under specialists' investigations, both because of its intrinsic interest and because several longstanding classical problems still remain unsolved.

In this context, the paper [2] makes use of the above mentioned duality to study the submodule $T^k\big(V^{\otimes m}\big)$ of all tensors of valence k in $V^{\otimes m}$; understanding the structure of this space was exactly one of the unsolved problems mentioned right now. The result in [2] is a description of $T^k\big(V^{\otimes m}\big)$ as a representation, for the Brauer algebra, induced from a simpler one. As a byproduct this description — essentially combinatorial — provides a new proof of Littlewood's restriction formula (for m big enough) which describes the restriction to $SP(V)$, or to $O(V)$, of an irreducible $GL(V)$–representation. The analysis and results of this work are improved in [5], where a more subtle combinatorial description of the

Brauer algebra and of its indecomposable modules yields a new proof of a stronger version of Littlewood's restriction formula: in particular, the original result for the orthogonal group is improved in this work, for the first time since Littlewood's article of 1944. With the like techniques, in [26] a great part of the radical of the Brauer algebra is described, and similarly for its indecomposable modules.

Supergroups: In classical geometry, the symmetry groups of interest are the Lie groups (in the differential setup) or the algebraic groups (in the algebraic framework). In supergeometry, these are replaced by Lie supergroups or algebraic supergroups: objects of both types can be defined with the language of topology (which better stress the underlying aspects of classical geometry), or in terms of functors of points (which better serves a wider generalizations). These supergroups are in close relation with Lie superalgebras via the super-analogue of Lie's theorems relating Lie groups and Lie algebras in the classical setting. Nevertheless, the study of Lie superalgebras is somehow easier - and it has been pushed forward much more - than that of supergroups. In particular, the classification (and structure theory) of supergroups is much less advanced than the Lie superalgebras one: indeed, even the construction of examples is somewhat more problematic.

In this context, with the papers [29], [30], [31], [32], [33] and [31-Cor] one moves a step forward in the direction of a (sort of) classification program for "simple" algebraic supergroups (roughly speaking, say). Indeed, these works provide an existence theorem for all connected algebraic supergroups whose tangent Lie superalgebra is simple. We recall that such Lie superalgebras (simple, finite-dimensional), whose classification is well-known, split into two classes: those of classical type — which are the superanalogue (in a sense), of simple finite-dimensional complex Lie algebras, or of affine complex Kac-Moody algebras — and those of Cartan type.

In detail, in the works [29] and [33] one obtains such an existence result for the classical case, by means of a direct, concrete construction, which mimics the classical one by Chevalley that provides all connected simple algebraic groups whose tangent Lie algebra is semisimple. Indeed, one starts with a classical Lie superalgebra and a simple representation of it: the desired algebraic supergroups are then given as subgroups (in the general linear supergroup over the representation space) generated by "one-parameter supersubgroups" attached to root vectors in the Lie superalgebra itself. In particular, this construction provides a unifying approach to most of the algebraic supergroups already known in literature; in addition, it yields an explicit recipe to produce new examples. Moreover, these supergroups are built in as "super group-schemes" over $\mathbf{Z}$. On the other hand in [30] — proceedings of a conference — one gives a short, thought-out presentation of this very construction, and in addition one shows also how one can extend the just described method to other contexts.

Some particular "Chevalley supergroups" (namely those of type $D(2,1;a)$, now considered as complex Lie supergroups) are investigated again in [38]. Roughly speaking, the Lie superalgebras of type $D(2,1;a)$ form a one-parameter family whose elements are (Lie superalgebras that are) simple for all but a finite number of values of the parameter: in this work we show that the construction of the associated Lie supergroups also makes sense for those "singular" values of the parameter, leading to non-simple supergroups that we describe in some detail. One can realize this in different ways (that yield different results in the singular case), five of these are presented in detail; while doing this, we also compare the approach by Scheunert with that by Kac (more largely followed in literature).

As a next step, in [32] one proves sort of a converse result of those in [29]: namely, one shows that every connected algebraic supergroup whose tangent Lie superalgebra is classical is necessarily isomorphic to a Chevalley supergroup of the type considered in [29].

With largely similar techniques and strategy, in [31] one proves a similar existence and uniqueness — up to isomorphisms — theorem for (connected) algebraic supergroups whose tangent Lie superalgebra is of Cartan type; in particular, the case of type $W(n)$ is treated a bit more in detail. The same topic is treated again in [31-Cor], where a mistake in [31] is amended and a key step of the main construction is clarified more in depth.

In [36] I dwell on the more general problem of studying an (affine) algebraic supergroup via its associated super Harish-Chandra pair — namely, the datum of its underlying classical algebraic group and its tangent Lie superalgebra. This is a key theme in supergroup theory, which is addressed by several authors in different ways; I myself present yet another approach, tightly related with the existence of "global splittings" for (affine) supergroups. Roughly, for a supervariety this is the property to split into direct product of a classical (algebraic) variety and a totally odd supervariety — in short, a global factorization of type "$\underline{\mathit{even}} \times \underline{\mathit{odd}}$": although this does not necessarily hold for supervarieties in general (it always works locally, but globally may fail), it is true instead — under mild conditions — for affine supergroups. The parallel question is dealt with in [39] and [E3] for Lie supergroups (of either real smooth, real analytic or complex holomorphic type) instead of algebraic supergroups: namely, the equivalence of Lie supergroups and super Harish-Chandra pairs is proved giving two new functorial recipes to construct, out of a given super Harish-Chandra pair, a suitably devised Lie supergroup. One of these recipes is (essentially) the same as in [36], but adapted to the differential setup (with quite a few critical technicalities to take care of); the other one instead is new — and might, conversely, be adapted to the algebro-geometrical framework as well.

In [43] we undertake the study of real forms — and in particular *compact* ones (in a suitable sense) — of complex Lie superalgebras and supergroups, showing that the situation is actually richer than in the classical case: indeed, besides the obvious generalization of the notion of real form (from the classical case to the super case) — called "standard" — there exists also a second one — called "graded" — which has no direct classical counterpart. In this sense, moving to the super context reveals in fact an unexpectedly richer situation. The study of symmetric pairs (and the associated symmetric superspaces) performed in [45] is also somehow related with all this.

On a different note, [46] is instead a somewhat "eccentric", standalone work, which deals with geometric quantization, in a super-Kähler setup, for the abelian Lie supergroups.

Finally, in [49] we work with yet another different topic: namely, we introduce a special class of Lie superalgebras (possibly with trivial super structure, hence just simply Lie bialgebras), which are called "GaGa algebras", characterized by having a (even) bi-Abelian subsuperbialgebra which acts and coacts diagonally, so that the superbialgebra itself splits into a direct some of remarkable subspaces (namely "weight & coweight spaces"). For these peculiar Lie superbialgebras one can find a very precise combinatorial description ("à la Kac–Moody", say), and their theory of deformations — by twist or by 2–cocycle — is highly interesting: both these lines of research are introduced and developed in this work. In addition, we show how several well-known classes of Lie superbialgebras fall within this framework, hence they can be treated as particular examples (non trivial!) of

GaGa algebras which “exist in nature”, and so motivate the introduction of a new, general notion encompassing and expanding all these specific examples.

Quantum Groups: Quantum Groups are algebraic deformations, in the category of Hopf algebras, either of universal enveloping algebras of Lie algebras — hence they are called *quantized universal enveloping algebras (QUEA in the sequel)* — or of function algebras of algebraic groups or Lie groups — then they are called *quantized function algebras (QFA in the sequel)*. Introduced in 1985 as “quantum symmetries”, they later prove themselves very interesting also in the study of algebraic groups in positive characteristic and their representations, or for the theory of knot (and link) invariants and many other topics. Moreover, they are intrinsically related to the geometric theory that one gets from them as semiclassical limits or, conversely, which they quantize: namely, the theory of Poisson (Lie or algebraic) groups and of Lie bialgebras, and more in general the geometry of Poisson varieties.

My contributions in this field divide into four main trends.

QFA=QUEA: The *“equivalence” between QUEAs and QFAs* is a first trend.

In [0] and [3], starting from the most famous quantum groups, built upon semisimple Lie algebras, with standard Poisson structure on the associated groups, I introduce quantum groups for the dual Poisson groups: the data I start from now are the QUEAs by Jimbo (and Drinfeld) and their integral forms, both restricted and unrestricted. Taking the first or the second ones the corresponding semiclassic limits respectively are universal enveloping algebras of Lie bialgebras or function algebras of Poisson groups: this is the basic idea, which applies as well to the case of Hopf algebras dual to the QUEAs I was starting from, thus yielding quantum groups “duals” to the above ones. The analogue of this work for affine Kac-Moody algebras is done in [7]: here the key tool is a theorem of Poincaré-Birkhoff-Witt (PBW) type for restricted integral forms of the affine QUEAs; such a result is stated and proved in [6].

In [4] I perform, dually, a similar construction to that of [0] and [3], yet more concrete; but now I start from the QFAs associated to $SL(n)$ or $GL(n)$, for which it is available a well-known presentation by generators and relations by means of “q–matrices”; another result in this paper is a PBW theorem for the QFA over $SL(n)$. All these results then get improved in [23] and [24].

For the $SL(n)$ case, looking at the corresponding QUEA as a QFA leads to find an alternative presentation: this is the content of [19], where such a presentation is given in terms of “q–matrices”. In particular, this yields an alternative approach to the well-known presentation by L–operators due to Faddeev, Reshetikhin and Takhtajan.

A further expansion of some aspects of [4] is the article [25]: in it some theorems of Poincaré-Birkhoff-Witt type are proved for the QFA associated to $Mat(n)$, to $GL(n)$ or to $SL(n)$, and for their specializations at roots of 1. As a corollary, one obtains also some interesting results about the structure of Frobenius algebra for these QFA at roots of 1.

Finally, another development of all these constructions is performed in [41], where “multiparametric” QUEAs are taken into account, along with their integral forms and their specializations at roots of 1, with results analogous to those already known for the uniparametric case.

QDP: The *quantum duality principle*, or QDP in the sequel, is the guiding idea of the second trend, and explains the results of the first one. In the original formulation by Drinfeld, the QDP provides an category equivalence between QUEAs and QFAs, for quantum groups defined (as Hopf algebras) over $\mathbf{k}[[\hbar]]$, where $\mathbf{k}$ is a field, and topologically complete: in [12] I give a complete and rigorous proof — the first one in literature — of this result. In [E1] and [20] instead I push forward this idea, formulating a much stronger version of the QDP for Hopf algebras defined over very general rings and without additional topological assumptions. Namely, I prove that Drinfeld's recipes do define two endofunctors of the category of such Hopf algebras, which realize a Galois correspondence in which these functors have as images the subcategory of the QUEAs and of the QFAs respectively; moreover, QUEAs and QFAs are exactly the subcategories of those objects which are fixed by the composition of the two functors. Of course, as the contexts are different the techniques which are involved in [12] and in [E1], [20] are rather different.

While [E1] is a widely expanded essay, enriched with several examples and applications, [20] is the journal article which treats just the main, central result of [E1], namely the theorem expressing the stronger version of the QDP explained above. Both [9] and [11] (proceedings of conferences) are short versions of [E1], each one enriched with an original example. On the other hand, [E2] instead (notes for a summer school) is a survey of the results in [E1], explained by means of several explicit examples and applications. The general discussion is completed with [47], in which we analyze the interplay between the QDP and the (Hopf theoretical) deformation procedures of QUEAs by twist and by 2-cocycle.

A remarkable case of QDP in an infinite-dimensional setup is studied in [42], where one considers a "continuous" analogue of the (polynomiale and/or formal) QUEA associated with a Kac-Moody algebra. In this context one cannot apply directly the general theory, but a direct analysis leads to a final result which much like the same as the one in the finite-dimensional case.

More in general, a direct application of the QDP to Hopf algebras defined over a field leads to the crystal duality principle, or CDP in short. This can also be obtained by classical means — i.e., without involving quantum groups — so it reads like a chapter of classical Hopf algebra theory. The works dealing with these are [15], [16] and [17]: for more details, see section *CDP* in "Hopf Algebras and related structures" below.

A further development is [18], where we state and prove a QDP for homogeneous spaces, or for the corresponding subgroups. As an application we compute an explicit quantization of an important Poisson structure on the space of Stokes matrices; a shorter, seminal version of this work is [21], where some other applications and examples are presented. Moreover, a version of this work in terms of *global* quantum groups is developed in [35], where in addition we consider different versions of "quantization" for subgroups; in this way also non-coisotropic subgroups can be taken into account, but our results show that in the end the coisotropic ones necessarily play a key role. Finally, these ideas are extended to the context of *projective* homogeneous spaces, studying the example of Grassmann varieties in [28] and the general case in [27].

In yet another direction, studying deformations (by twist or by 2–cocycle, in the sense of Hopf algebra theory) of quantum groups, in [47] we prove that the QDP is the underlying reason for which it is possible to introduce a construction of "deformations by *polar twist*, respectively by *polar 2–cocycle*", for QUEAs, respectively for QFSHAs. This allows to extend in a non trivial way the usual, standard constructions of Hopf theory, thus introducing new tools for the deformation theory of quantum groups.

Finally, in [34] we explore the possibility to extend all this lot of ideas to the framework of "quantum groupoids", i.e. quantizations of bialgebroids: here the notion of bialgebroid is a suitable generalization of that of bialgebra, we deal with quantization in a formal sense and, at the semiclassical level, Lie (bi)algebras are replaced by Lie-Rinehart (bi)algebras - sometimes called just "Lie (bi)algebroids". In particular, we develop a convenient form of QDP for these objects (also showing this "at work" in a specific example).

R-MAT: *R–matrices and braidings* are the core of the third trend. The notion of R–matrix for a QUEA is the quantization of the notion of classical r–matrix for a Lie bialgebra, which corresponds to consider those Lie bialgebras whose Lie cobracket be a particular coboundary. More in general, the Hopf algebras endowed with an R–matrix correspond, via Tannaka-Krein duality and associated reconstruction theorems, to the braided monoidal categories, i.e. those endowed with an analogue of the tensor product and of the twist ("exchange of factors") automorphism for it. It springs out of this the interest of this algebras in quantum and conformal field theories, as well as in topology for the construction of link invariants and of 3-variety invariants. One gets a weaker notion by substituting the R–matrix with a suitable automorphism of the Hopf algebra, which is called *braiding*.

In my work upon this topic I applied the QDP (see above) to those QUEA endowed with an R–matrix (the so-called "quasitriangular" ones): the main result is that, given a Lie bialgebra endowed with a classical r–matrix (also called "quasitriangular"), one finds a geometrical counterpart of such an r–matrix for *dual* formal Poisson group, thus explaining the relationship between r–matrices and Poisson duality (among Poisson groups).

In [1], given a QUEA over a semisimple Lie algebra, and its standard R–matrix, sorting out of the QUEA a QFA (after the QDP, like in [E1] or in [E2] or in [22]) I prove that the adjoint action of the R–matrix does specialize to a distinguished automorphism on this QFA. Moreover, the semiclassical limit of the latter is a birational automorphism of the dual Poisson group, and more precisely a braiding, in geometrical sense; thus I widely generalize a result proved by Reshetikhin for $SL(2)$. All this I later extend to the case of Kac-Moody algebras in [8]. In [10] we perform a further generalization, proving that a similar result does hold for any quasitriangular QUEA: here we apply the QDP as formulated in [12], i.e. for topological quantum groups directly using the general definition of the functor $QUEA \longrightarrow QFA$ rather than an explicit description as one does in [1] and [8].

In [13] we compare the results of [1] with those of Weinstein and Xu, who make an analogous braiding on the dual of a quasitriangular Poisson group by means of purely geometrical methods. Our first result is that both braidings are "infinitesimally trivial". The second is that in the case of $SL(2)$ these braidings do coincide: the proof comes out of an explicit description of both of them, via direct computation.

Finally in [14] we show that, given a formal quasitriangular Poisson group G, with classical r–matrix r, a braiding associated to it on the dual formal Poisson group G^* is *unique*: in particular the one in [13] and that in Weinstein and Xu always coincide. Furthermore we make precise the nature of such a braiding, proving that it is Hamiltonian, corresponding to some function ρ on G^*, which in turn is a "lifting" of r from the cotangent Lie bialgebra of G^* to the function algebra of G^* itself. We provide two independent construction of such a lifting ρ: in the first one, ρ is given as semiclassical limit of the "logarithm" of a quantum R–matrix which quantizes r ; in the second one, we make out ρ directly as a lifting of r by iterated approximations, where the possibility of performing the n–th step is proved by cohomological means.

To finish with, in [47] we consider further constructions concerning R–matrices — along with their dual counterparts, namely ϱ–comatrices — which are standard in Hopf algebra theory. In fact, we prove that in the case of quantum groups (either QUEA or QFA, in the formal framework) they actually provide very precise results which involve once more the Quantum Duality Principle applied to the involved quantum group. Moreover, we also show how these constructions extend to a broader setup involving the more general notions of "polar–R–matrix" and "polar–ϱ–comatrix".

Multiparameter quantum grous and deformations: The *multiparameter* quantum groups are quantum groups — i.e. QUEAs or QFSHAs/QFAs — which depend on more than one parameter: among these parameters, however, only one has "quantum value", while the other ones have "geometrical value". In literature, people studied both those where the "geometrical parameters" affect the coalgebra structure and those which affect the algebra structure.

The first case I studied is that of multiparameter quantum groups which are dual to the semisimple ones where the structure of coalgebra was modified (by means of "geometrical" parameters): the results are presented in [0] and in [3], whiche show that in these dual quantum groups the geometrical parameters affect instead (dually!) the structure of algebra. In [41] instead we study multiparameter QUEAs (of *polinomial* type) for Kac–Moody algebras of symmetrizable type, in which the geometrical parameters affect the structure of algebra. This study is resumed and deepened in [40] and in [44], in terms of QUEAs of *formal* type, and then it is extended in [48] to the case of quantum *super*groups. In all these cases — including [0] and [3] as well — we study also the semiclassical limits of these structures, which are "multiparameter" Lie (super)bialgebras that are described in detail. Finally in [51] we extend this study to the case of QFAs for multiparameter quantum groups.

On the other hand, quantum groups are special type Hopf algebras, and as such they can be "deformed" so to obtain new Hopf algebras — and possibly, new quantum groups; such deformations are essentially of two types, by twist or by 2–cocycle, the first one change the structure of coalgebra, the second one that of algebra. In particular, by means of these constructions one can obtain multiparameter quantum groups, starting with uniparameter quantum groups and deforming them in a suitable way (via a particular twist or 2–cocycle). For instance, the results in [41] to a large extent are based upon this kind of construction. More in general, the link between multiparameter quantum groups and deformations is studied at length in [40], [44], [48] and [51]: in detail,

(a) in [40] and in [44] we study QUEAs for multiparameter quantum groups, and their deformations, proving in particular that deformations by twist or by 2–cocycle are ("morally") of the same nature, in particular they leave stable this class of multiparameter QUEAs;

(b) in [48] we extend the study made in [44] to the case of (formal) QUEAs for quantum multiparameter *super*groups, with totally similar results;

(c) in [51] we develop the analogous study to [40] and [44], but dedicated to the case of QFAs for multiparameter quantum groups.

Finally, the work [47] analyzes methodically the subject of the construction of deformations (by twist or by 2–cocycle) of quantum groups, and of their effect onto the semiclassical limit: the main result — inspired by the QDP — is that one can extend these procedures up to construct "deformations by *polar–twist*" for (formal) QUEAs and "deformations by *polar–2–cocycle*" for QFSHAs. This extends in a non-trivial way the standard constructions of Hopf theory, thus introducing new tools for deformation theory of quantum groups, as well as for their semiclassical limits.

Hopf Algebras and related structures: The theory of Hopf algebras is a classical topic which gained new interest in the last twenty years, mainly for its interplay with such diverse fields as quantum groups, low-dimensional topology, tensor categories, supergeometry, etc.

My contributions in this field divide into three main trends.

CDP: The *crystal duality principle*, or CDP in short, is an important corollary of the QDP, which is obtained as an application of the latter to Hopf algebras defined over a field and for which scalars are extended to polynomials over that field. Nevertheless, such a result can be achieved almost entirely by means of techniques and tools of the "classical", i.e. "non quantum", theory of Hopf algebras over a field: in this way one gives life to a new chapter of the "standard" theory, in which to any Hopf algebra (which can be thought of as a generalized symmetry) one associates Poisson groups and Lie bialgebras (which are geometrical symmetries). This approach of "classical" type is realized in [17]. A short version of such a work is [15] (proceedings of a conference). Instead, [16] is the explicit study in detail of an important example, a Hopf algebra built out of the Nottingham group of formal series of degree 1, with the composition product. This is just but one among several examples of Hopf algebras built upon combinatorial data (graphs, trees, Feynman diagrams, etc.) which naturally show up in (co)homology, non-commutative geometry and quantum physics; so it is highly instructive as a "toy model" of more general situations.

Quasitriangular structures (and generalizations): A very special class of Hopf algebras is that for which — roughly speaking — the lack of cocommutativity is somehow "under control". This idea is encoded in the notion of *quasitriangular* Hopf algebras and in its various generalizations. I studied this topic in a series of papers — [1], [8], [10], [13], [14] and [47] — in which the Hopf algebras under exam are always quantum groups: for further details, see section *R-MAT* in "Quantum Groups" above.

Generalizations (quasi-Hopf algebras, Hopf superalgebras, etc.): Hopf algebras have been generalized in several ways: among these, I consider the cases of *quasi-Hopf algebras* and of *Hopf superalgebras*. In the first case, one is weakening the coassociativity axiom; in the second, one is considering Hopf algebras in the category of *super (i.e.,* $\mathbf{Z}_2$*–graded) vector spaces* — or supermodules over a ring — so that tensor products must be handled in a different way.

The study of *quasi-Hopf algebras* became very important due to Drinfeld's works in the second half of the '80s of last century. The main ingredient in this study is the notion of "associator", an object which then turns useful in other contexts as well (for instance, to solve the general problem of the quantization of Lie bialgebras). As a matter of fact, to date the sole associator which is really known is the so-called *KZ associator*, obtained as solution of the Knizhnik-Zamolodchikov differential equation (with respect to the same name connection on $\mathbf{C}^n$), for which it was known — explicitly — an additive formula only. In [20] we provide instead an explicit formula for the *logarithm* of this associator (as a special application of a more general result), in terms of multiple ζ–functions.

As to Hopf superalgebras, the commutative ones (in a "super sense") have a geometrical meaning: namely, their spectra are the so-called affine algebraic *supergroups*, just as classically the affine algebraic groups are the spectra of commutative Hopf algebras. My main contributions to this topic are in [29], [30], [31], [32], [33], [36], [31-Cor], [38], [39], [E3] e [43], whose content is explained in detail above — see the section *Supergroups* in "Algebraic Groups, Representations and related topics".

Finally, an important extension of the notion of Hopf algebra (actually, of bialgebra indeed) is that of *bialgebroid*: this has shown increasing importance in several contexts, e.g. in non-commutative geometry. I start investigating this in [34], which is devoted to study quantizations (in formal sense) of bialgebroids. In particular, here we study the functors providing linear duality among (quantum) bialgebroids — something more or less already known — and we introduce new, suitable functors "à la Drinfeld" which establish a QDP for quantum bialgebroids — the (main) original contribution of this paper (see also the section *QDP* in "Quantum Groups" here above). These topics are further developed in [50], where the focus is set upon the subclass of "action bialgebroids", and among these in particular onto "action quantum groupoids".

Still in this context one has also [37], that is specifically devoted to the study of duality for bialgebroids with suitable additional structure.

— PUBLICATIONS —

Works in progress

[51] — G. A. García, F. Gavarini, *"Multiparameter quantum function algebras and their deformations"*

[50] — S. Chemla, F. Gavarini, N. Kowalzig, *"Duality functors for action quantum groupoids"*

[49] — G. A. García, F. Gavarini, *"GaGa algebras"*

Preprints

[48] — G. A. García, F. Gavarini, M. Paolini, *"Multiparameter quantum supergroups, deformations and specializations"* — preprint `http://arxiv.org/abs/2410.22549 [math.QA]` (2024), 64 pages;

[47] — G. A. García, F. Gavarini, *"Quantum group deformations and quantum R–(co)matrices vs. Quantum Duality Principle"*, 58 pages (2024) — **N.B.:** extended version in `http://arxiv.org/abs/2403.15096 [math.QA]` (2024), 72 pages

[**46**] — M.-K. Chuah, F. Gavarini, *"Super Kähler structures on the complex Abelian Lie supergroups"* — preprint `http://arxiv.org/abs/2312.00444 [math.DG]` (2023), 29 pages;

[**45**] — M.-K. Chuah, R. Fioresi, F. Gavarini, *"Admissible Systems and Graded Hermitian Superspaces"*, submitted (2020), 15 pages.

Journal articles, proceedings, etc.

[**44**] — G. A. García, F. Gavarini *"Formal multiparameter quantum groups, deformations and specializations"*, Annales de l'Institut Fourier (to appear), 117 pages — preprint `http://arxiv.org/abs/2203.11023 [math.QA]` (2022);

[**43**] — R. Fioresi, F. Gavarini, *"Real forms of complex Lie superalgebras and supergroups"*, Communications in Mathematical Physics **397** (2023), no. 2, 937–965 — **DOI:** `10.1007/s00220-022-04502-x` ;

[**42**] — F. Gavarini, *"Quantum duality principle for quantum continuous Kac-Moody algebras"*, Journal of Lie Theory **32** (2022), no. 2, 839–862;

[**41**] — G. A. García, F. Gavarini, *"Multiparameter quantum groups at roots of unity"*, Journal of Noncommutative Geometry **16** (2022), no. 3, 839–926 — **DOI:** `10.4171/JNCG/471` ;

[**40**] — G. A. García, F. Gavarini, *"Twisted deformations vs. cocycle deformations for quantum groups"*, Communications in Contemporary Mathematics **23** (2021), no. 8 - 2050084 (56 pages) — **DOI:** `10.1142/S0219199720500844` ;

[**39**] — F. Gavarini, *"A new equivalence between super Harish-Chandra pairs and Lie supergroups"*, Pacific Journal of Mathematics **306** (2020), no. 2, 451–485 — **DOI:** `10.2140/pjm.2020.306.451` ;

[**38**] — K. Iohara, F. Gavarini, *"Singular degenerations of Lie supergroups of type $D(2,1;a)$"*, SIGMA **14** (2018), 137, 36 pages — see `https://www.emis.de/journals/SIGMA/2018/137/` — **DOI:** `10.3842/SIGMA.2018.137` ;

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