Annihilating Polynomials of Excellent Quadratic Forms

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May 21, 2008

Let K be a field. Consider the semi-ring $\widehat{W}^+(K)$ of isomtry classes of quadratic forms over K. We obtain the *Witt-Grothendieck ring* $\widehat{W}(K)$ by applying the Grothendieck construction to $\widehat{W}^+(K)$. Now the *Witt ring* $W(K)$ is the quotient of $\widehat{W}(K)$ by the principial ideal generated by the isometry class of the hyperbolic plane. Both $\widehat{W}(K)$ and $W(K)$ are integral, i.e. for every element x of $\widehat{W}(K)$ or $W(K)$ there exists a polynomial $P \in \mathbb{Z}[X]$ such that $P(x) = 0$. The study of *annihilating polynomials* of quadratic forms is quite young. It was initiated by David Lewis in 1987, when he showed that for $n \in \mathbb{N}_0$ the polynomial

$$P_n \ := \ (X-n)(X-n+2)\cdots(X+n-2)(X+n) \ \in \ \mathbb{Z}[X]$$

annihilates the isometry class of every n-dimensional quadratic form over any field. Since then some effort has been invested into identifying classes of quadratic forms that allow annihilating polynomials of lower degree.

In this talk we will study annihilating polynomials of *excellent* quadratic forms. Excellent forms naturally occur in the theory of generic splitting of quadratic forms, since excellent forms over a field K can be characterized by the fact that all their higher anisotropic kernels are defined over K. Let φ be an n-dimensional excellent form over K, and let $n_0 = n, n_1, \ldots, n_h \in \mathbb{N}_0$ be the dimensions of the higher anisotropic kernels of φ. We will show that

$$E_n \ := \ \begin{cases} X(X^2-n_{h-1}^2)\cdots(X^2-n_1^2)(X^2-n^2) & \text{for } n \text{ even,} \\ (X^2-1)(X^2-n_{h-1}^2)\cdots(X^2-n_1^2)(X^2-n^2) & \text{for } n \text{ odd,} \end{cases}$$

annihilates the isometry class of any n-dimensional excellent form over any field. As is the case for P_n it can be shown that there exists a field K and an excellent form φ over K such that E_n is the unique "minimal" annihilating polynomial of the class of φ in both $\widehat{W}(K)$ and $W(K)$.