COURSE: Controllability of Equations of Fluid Dynamics

In this course we consider the problems of exact controllability of the fluid flow with locally distributed or boundary control. We pay the special attention to incompressible fluid modeled by the Navier-Stokes system or by the Euler equations.

The typical controllability problem, which we are going to discuss in the case of the Navier-Stokes system is the following. We consider the fluid flow in the domain Ω. Let $\mathbf{v}(t, \mathbf{x})$ be velocity of the fluid at the point $\mathbf{x}$ at moment $\mathbf{x}$ and $\mathbf{f}(t, \mathbf{x})$ be the density of external forces and $p(t, \mathbf{x})$ be the pressure.

$$\partial \mathbf{v} \partial t - \Delta \mathbf{v} - (\mathbf{v}, \nabla)\mathbf{v} = \nabla p - \chi_\omega \mathbf{u} \quad in\ \Omega,$$

$$B\mathbf{v} = 0, div\, \mathbf{v} = 0,$$

$$\mathbf{v}(0, \cdot) = \mathbf{v}_0.$$

Here B is the operator of the boundary conditions, say $b\mathbf{v}|_{\partial\Omega} = 0$, $\mathbf{v}_0$ is given velocity field, $\omega \subset \Omega$ is the arbitrary but fixed sub domain of Ω χ_ω is characteristic function of ω and $\mathbf{u}$ is the locally distributed control. At the moment t we are starting from the velocity field v_0. Is it possible to find a locally distributed control $\mathbf{u}$ such that at the moment T we have

$$\mathbf{v}(T, \cdot) = \mathbf{v}_1.$$

Here $\mathbf{v}_1$ is another velocity field called the target function.

The machinery used in this course includes the Sobolev spaces, energy and Carleman estimates, pseudo differential operators.