HOLOMORPHIC DYNAMICS

PhD course – Dipartimento di Matematica
Università di Roma "Tor Vergata"

Leandro Arosio

This course is an introduction to the theory of holomorphic automorphisms of the complex space $\mathbb{C}^n$, with particular emphasis on Andersén-Lempert theory. It is well-known that for $n = 1$ every holomorphic automorphism is affine: $z \mapsto az + b$, $a \neq 0$. The situation is completely different in several complex variables, indeed $\mathsf{aut}(C^n)$ is a huge and complicated group for all $n \geq 2$. Here follows a list of topics that will be considered during the course.

- Fatou-Bieberbach domains. If $n \geq 2$ there exists proper domains of $\mathbb{C}^n$ biholomorphic to $\mathbb{C}^n$. This kind of domains appear in particular as basins of an attractive fixed point for an automorphism of $\mathbb{C}^n$ which admits at least another fixed point. The proof is based on the Poincaré-Dulac method.
- The Andersén-Lempert theorem. If $n \geq 2$ and f is a biholomorphism between a starlike domain and a Runge domain of $\mathbb{C}^n$, then f is the uniform limit on compact subsets of automorphisms of $\mathbb{C}^n$.
- Forstneric-Weickert theorem. Let $P\colon \mathbb{C}^n \to \mathbb{C}^n$ be a polynomial mapping with $dP(0)$ invertible. Let d be the biggest degree of its components. Then there exists an automorphism Ψ of $\mathbb{C}^n$ such that the d-jet of Ψ at 0 equals P.
- Construction of a non-Runge Fatou-Bieberbach domain and of a non-Stein long $\mathbb{C}^2$.