General Relativity

Roma "Tor Vergata" PhD School 2018

1. Historical introduction. Introduction to the formalism of special relativity. Covariance of the equations of the physics.
2. Fundamental points of the Einstein's theory of gravitation.
3. Introduction to Riemannian geometry: mean and Gaussian curvatures, surface's metric, covariant derivative, Christoffell's symbols, Gauss theorem. Curvature tensor. Parallel transport equation. Geodesic equation. Prallelizable manifold and ciclic transport. Riemann curvature tensor. Curvature invariants. Sectional curvature and Schur's theorem.
4. Energy-momentum tensor. Equations of the General Relativity. General covariance and the meaning of the coordinates.
5. Reference frame, coordinates adapted to a given reference frame. Quasi-product structure of the space-time.
6. Isometries and Killing vectors.
7. Hints on the causal structure of the space-time. Global hyperbolicity. Some singularity theorems. Raychaudurhi equation.
8. Exact solutions and their interpretation. Internal and vacuum solutions. Darmois-Israel junction conditions. The Schwarzschild and Kerr solutions. Black holes, event horizons, Kruskal coordinates.
9. Bekenstein argument, black hole entropy and its thermodynamics. Black hole evaporation.
10. Cosmological solutions and their physical interpretation. Friedmann-Lemaitre universes. Big bang. Dark energy.
11. Methods to generate exact solutions: Ernst method for stationary axisymmetric solutions and Ehlers theorem for dust-rotating solutions.
12. Formulation of the initial value problem in General relativity and characteristic surfaces.
13. Hints of perturbation theory and gravitational waves. Development of a tensor in a basis of spherical tensorial harmonic. Regge-Wheeler and Zerilli equations.

Bibliography

1. R. Wald, *General Relativity*, The University of Chicago Press, Chicago (1984).

2. S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Space-time*, Cambridge University press (1073).

3. G. Ferrarese, *Lezioni di Relativitá Generale*, Pitagora Editrice (1994).

4. S. Chandrasekhar, *The mathematical Theory of Black Holes*, Oxford University Press, New York Oxford (1992).