Introduction to non-commutative analysis

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In the framework of non-commutative geometry the singular (Dixmier) traces, originally introduced by J. Dixmier in 1966, have become an indispensable tool. These traces are defined via dilation invariant extended limits on the space of bounded measurable functions. As a part of this talk we discuss in details the formulae relating Dixmier traces and zeta-function residues. These formulae, among others important results in non-commutative geometry (e.g. Connes Character Theorem, relation between Dixmier traces and heat functionals), are established under various additional conditions on these extended limits.

Every such condition distinguishes a subclass of Dixmier traces. In the present talk we also discuss the relation between these classes and an important concept of measurable operators (with respect to these subclasses) introduced in non-commutative geometry by A. Connes in 1988. In most cases we provide new characterisations of measurability and definitive description of classes of measurable operators.