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Metric Geometry and Curvature of Domains in $\mathbb{C}^n$

I. Generalities on metric spaces

I.1. The Poincaré metric and generalizations

I.2. The Hopf-Rinow Theorem for Length Spaces
Let X be a complete, locally compact, length space. Then X is a proper geodesic space.

II. Gromov hyperbolic spaces

II.1. Definitions and examples

II.2. The Bonk-Schramm embedding Theorem
Isometric embedding of any Gromov hyperbolic metric space in a complete, geodesic, Gromov hyperbolic metric space.

II.3. Geodesic stability
(X,d) geodesic Gromov space, $A \geq 1$, $B \geq 0$. Then there exists $r > 0$ such that every (A,B)-quasi-geodesic $\gamma : [a,b] \to X$, joining p to q, satisfies : $d_{Haus}(\gamma([a,b]),[p,q]) \leq r$.

II.4. The boundary of a Gromov hyperbolic space
- Existence of geodesic rays and geodesic lines in proper, geodesic, Gromov hyperbolic spaces
- *The Visibility Theorem.* Given $x \neq y \in \partial_G X$, where X is Gromov hyperbolic, proper, geodesic, there is $K \subset\subset X$ such that every geodesic line joining x to y intersects K.
- Geodesic lines stability
- **The Bonk-Schramm filling Theorem** : construction of a Gromov hyperbolic metric space $Cone(Z)$ whose boundary is a given bounded metric space Z. Link between $Cone(\partial X)$ and X when X is a Gromov hyperbolic metric space.

II.5. Gromov hyperbolicity of some complete Kobayashi hyperbolic domains in $\mathbb{C}^n$
- Z.Balogh-M.Bonk's result : If D is strongly pseudoconvex, then (D,K_D) is Gromov hyperbolic and $\partial_G D$ can be identified with ∂D.
- The Squeezing function and associated results

III. Curvature of Kähler metrics

III.1. Definitions and links (from holomorphic sectional curvature to sectional curvature)

III.2. Examples

III.3. The Yau-Ahlfors-Schwarz Lemma and application
Theorem. (M,g) complete, Kähler, $Ric(g) \geq -a$ $(a \geq 0)$, (N,h) Hermitian, $Bisect(h) \leq -b < 0$. If $f : M \to N$ holomorphic, then $f^*h \leq (a/b)g$.

III.4. Non-existence of a complete Kähler metric with pinched negative holomorphic bi-sectional curvature on a polydisk (Yang's Theorem)

III.5. Bounded geometry and deformation of a complete Kähler metric with stable pinching condition