Feedback Control of the instationary incompressible Navier-Stokes equations

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The main objective of these lectures is to introduce the audience to recent advances in the mathematical analysis of the active control of the Navier-Stokes equations.

1. Mathematical Analysis of the Navier-Stokes equations

1.1. The Stokes operator. Analitycity of the Stokes semigroup.

1.2. Parabolic estimates and application to the Stokes and the Oseen equations.

1.3. The Stokes and the Oseen instationary equations with non homogeneous boundary conditions.

1.4. The instationary Navier-Stokes equations with non homogeneous boundary conditions.

2. Optimal control problems for the Navier-Stokes equations with a finite time horizon

We shall study the case of a distributed and a boundary control of the 2D Navier-Stokes equations. We shall study the existence of solutions to some control problems and derive optimality conditions. The following problems will be considered: Minimization of the drag, of the vorticity, of a tracking type functional.

3. Linear-Quadratic control problems of the Oseen equations – Infinite time horizon control problems

3.1. Feedback formulation of the optimal control of the Oseen equations with a finite time horizon: Analysis of the corresponding Differential Riccati Equation for an internal control operator and a boundary control operator.

3.2. Feedback formulation of the optimal control of the Oseen equations with a infinite time horizon: Analysis of the corresponding Algebraic Riccati Equation for an internal control operator and a boundary control operator.

4. Feedback Stabilization of the Navier-Stokes equations

4.1. The Lyapunov function method in the 2D and the 3D case: The local boundary stabilization of the Navier-Stokes equations by a linear feedback control law coming from a linearized problem requires some regularity of the optimal state. This regularity can be obtained by a particular choice of the cost functional. The local stabilization result is obtained by showing that some quadratic functional is a Lyapunov functional for the nonlinear closed loop system. This is the appraoch followed in [1] for an internal control and in [4] for a boundary control.

4.2. The smoothing observation operator method: The regularity necessary to deal with the nonlinear stabilization problem can be obtained by studying the optimality system of a nonhomogeneous control problem. This is the approach followed in [6] and [7].

4.3. Links with the Hamilton-Jacobi-Bellman theory: We shall prove the existence of regular local solutions to some Hamilton-Jacobi-Bellman equation, and we shall derive some nonlinear feedback law. The linear approximation of this feedback law corresponds to the feedback control determined in the previous section.

References

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