Proposal for a graduate course (30 hours)
University Roma II – Tor Vergata
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STEIN MANIFOLDS AND HOLOMORPHIC MAPPINGS

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Stein manifolds can be characterized as closed complex submanifolds of complex Euclidean spaces. Many analytic problems on such manifolds have only topological (homotopical, cohomological,...) obstructions, a phenomenon known as the *Oka principle*. The aim of this course is to survey the classical results and some recent developments on this rich subject.

SYLLABUS

Stein manifolds (a review). Definition and examples. Domains of holomorphy. Plurisubharmonic functions and pseudoconvex domains. Solvability of the $\overline{\partial}$-equation. Cartan's theorems A and B. Topological structure of Stein manifolds. Existence of Stein domains in complex manifolds. Holomorphic automorphisms of Euclidean spaces $\mathbb{C}^n$.

The Oka-Grauert theory. A review of some classical problems. Classification of principal fiber bundles and of complex vector bundles. The Oka properties and formulation of the main problems.

Existence theorems for holomorphic maps. Cartan-type splitting lemmas. Patching of holomorphic maps. Equivalence of Oka properties. Applications of sprays. Holomorphic sections of elliptic submersions. Optimal embedding and immersion theorems into Euclidean spaces. Hierarchy of holomorphic flexibility properties.

Holomorphic submersions and foliations. Noncritical holomorphic functions on Stein manifolds. Holomorphic submersions and a localization principle. Nonvanishing closed holomorphic 1-forms with prescribed periods. Existence theorems for nonsingular holomorphic foliations.

Handlebody constructions of Stein structures. Building Stein manifolds by surgery. Attaching handles along Legendrian submanifolds of strongly pseudoconvex boundaries. Extension of holomorphic maps across handles. Application of the gauge theory to Stein surfaces. Casson handles and exotic Stein surface structures on certain orientable four manifolds.