The Atiyah-Singer index theorem

October 11, 2018

Description

In this course we give an introduction to the heat kernel proof of the Atiyah-Singer index theorem for Dirac-type operators. Prerequisites: this approach to the index formula requires only basic notions of differential geometry and functional analysis. Interested students from every mathematical background are welcome to attend.

Program

1. Presentation, statement of the Hirzebruch signature formula, the Chern-Gauss-Bonnet formula, the Riemann-Roch-Grothendieck formula, vector bundles and connections.

2. Characteristic classes: basic Chern-Weil theory, Chern classes, Pontryagin classes, $L$-class, $\hat{A}$-class, Todd class.


4. Clifford modules and Dirac operators: basics of Clifford algebras and Spin geometry, Dirac operators.

5. Analysis of Dirac operators: the Bochner-Lichnerowicz-Weitzenboch formula for the square of a Dirac operator, application to positive scalar curvature obstructions on Spin manifolds, Sobolev spaces, elliptic regularity and Fredholmness of elliptic operators.


7. Traces: Hilbert-Schmidt operators, trace-class operators, Lidski’s theorem, gradings on Clifford modules, the canonical grading, basic Hodge theory (signature operator).
8. Traces and the index: supertraces and the McKean-Singer formula, relation of the index with the heat kernel expansion.

9. The local formula: the idea of Getzler and the Harmonic oscillator, local computation for Spin manifolds, Hirzebruch signature formula, Riemann-Roch-Grothendieck formula, Chern-Gauss-Bonnet formula

Selected references

