$$f(x) = 3x + \operatorname{arctg}\left(\frac{3x+3}{3x+2}\right)$$

$$\operatorname{dom} f = x \neq -2/3$$

$$\lim_{x \to +\infty} f = +\infty$$

$$f(x) = 3x + \underbrace{\operatorname{arctg} 1}_{= \pi/4} + o(1)$$

$y = 3x + \frac{\pi}{4}$ as. obl. a $+\infty$

$$\lim_{x \to -\infty} f = -\infty$$

$y = 3x + \frac{\pi}{4}$ obl. as. a $-\infty$

$$\lim_{x \to -\frac{2}{3}^+} f = 3\left(-\frac{2}{3}\right) + \frac{\pi}{2} + o(1) = -2 + \pi/2$$

$$\lim_{x \to -\frac{2}{3}^-} f(x) = 3\left(-\frac{2}{3}\right) - \frac{\pi}{2} + o(1)$$

$$= -2 - \pi/2$$

$x = -2/3$ punto di discont. di salto

-3

$$f' = 3 + \frac{1}{1 + \left(\frac{3x+3}{3x+2}\right)^2} \cdot \frac{\overset{\cancel{9x}+6-\cancel{9x}-9}{3(3x+2)-3(3x+3)}}{(3x+2)^2}$$

$$= 3\left(1 - \frac{1}{\underbrace{(3x+2)^2 + (3x+3)^2}_{\underbrace{9x^2+4+12x+9x^2+9+18x}_{18x^2+30x+13}}}\right)$$

$$= 3\,\frac{18x^2+30x+12}{18x^2+30x+13} = 18\,\frac{3x^2+5x+2}{18x^2+30x+13}$$

$3x^2+5x+2=0$ $\qquad x = \frac{-5 \pm \sqrt{25-24}}{6}$

$$= \frac{-5 \pm 1}{6} \; \begin{cases} -1 \\ -2/3 \end{cases}$$

$18x^2+30x+13 > 0$ toujours positif

$\Delta = 15^2 - 18 \times 13 = 225 - 234 < 0$

signe f' = signe $(3x^2+5x+2)$

$\downarrow$ sella

	-1		$-2/3$	
$+$		$-$		$+$
$\nearrow$		$\searrow$		$\nearrow$
	p.to max			

$$f'' = 18\,\frac{(6x+5)(18x^2+30x+13) - (36x+30)(3x^2+5x+2)}{(18x^2+30x+13)^2}$$

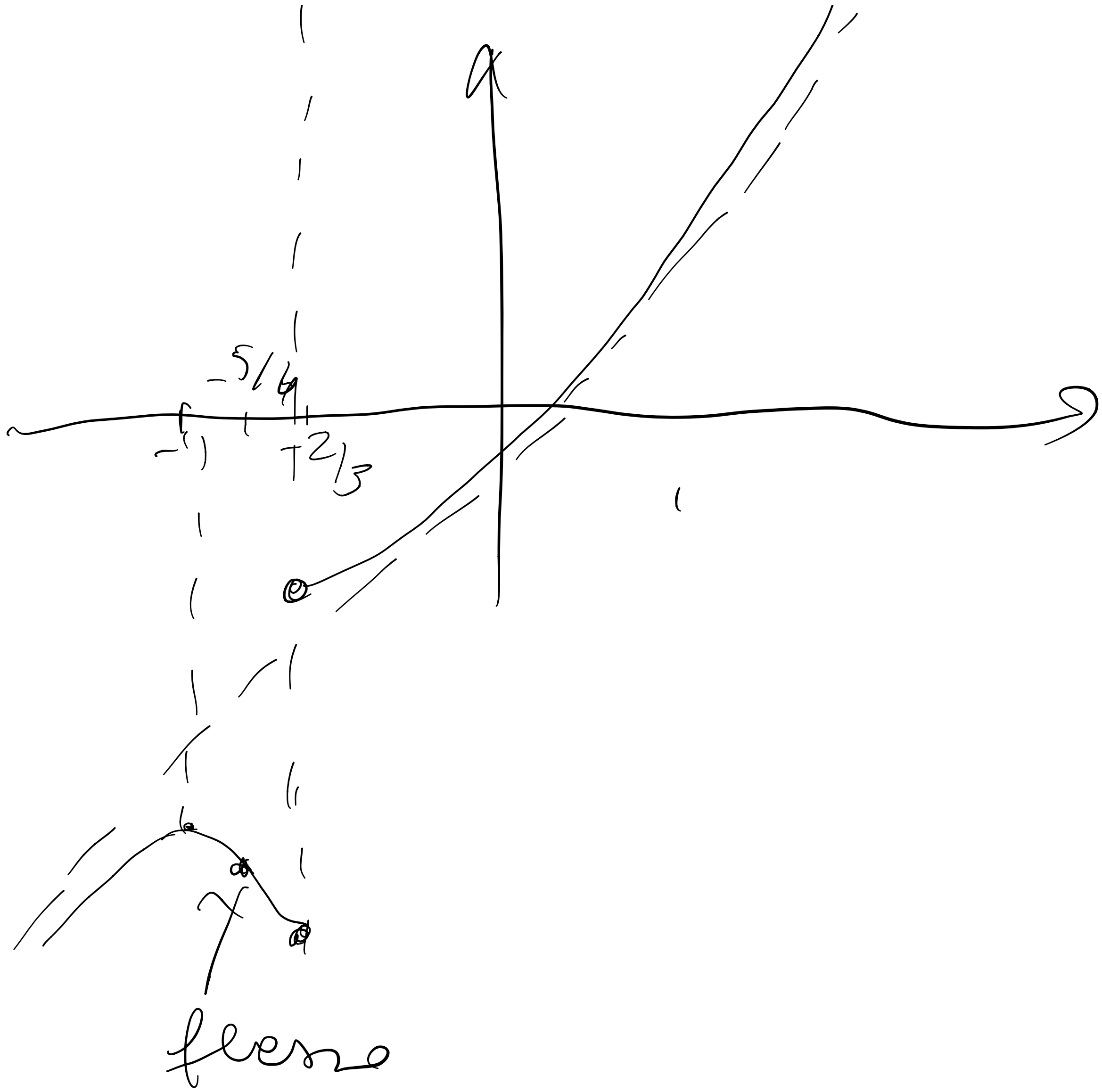
$$\Rightarrow \operatorname{sgn} f'' = \operatorname{sgn}\Big(\cancel{(18\cdot 6 - 36\cdot 3)}x^3 + \cancel{(180 + 18\cdot 5 - 36\cdot 5 - 3\cdot 30)}x^2 + (\cancel{150} + 6\cdot 13 - 72 - \cancel{60})x + 13\cdot 5 - 60\Big)$$

$$= \operatorname{sgn}(6x+5) \Rightarrow$$

$x > -5/6$ f conv.

$x < -5/6$ f conc.

$x = -5/6$ p.to di flesso


a
-5/4
-1
-2/3
1
flesso

$$\lim_{x\to 0} \frac{\cos x - 1 + \frac{1}{2}\operatorname{sen}(x^2)}{(\operatorname{tg} x)\left(e^x - 1 - \lg(1+x) - x^2\right)}$$

$$= \frac{\cancel{-\frac{1}{2}x^2} + \frac{1}{4!}x^4 + o(x^5) + \frac{1}{2}\left(\cancel{x^2} - \frac{1}{3!}x^6 + o(x^7)\right)}{\left(x + o(x)\right)\left(\cancel{x} + \cancel{\frac{1}{2}x^2} + \frac{1}{3!}x^3 + o(x^3) - \cancel{x} + \cancel{\frac{x^2}{2}} - \frac{x^3}{3} - \cancel{x^2}\right)}$$

$$= \frac{\frac{1}{4!}x^4 + o(x^5)}{x\left(\frac{1}{6} - \frac{1}{3}\right)x^3} = \frac{\frac{1}{4!}}{-\frac{1}{6}} = \frac{1}{24}(-6) = \boxed{-\frac{1}{4}}$$

$$\int_3^{+\infty} \frac{(\lg(x-2))^{\alpha+1}}{(e^{x-3}-1)^{1/2}}\,dx$$

$$f(x) = \frac{(\lg(x-2))^{\alpha+1}}{(e^{x-3}-1)^{1/2}} \underset{x\to+\infty}{=} \frac{(\lg x)^{\alpha+1}}{\sqrt{e^x}}$$

$$< \frac{1}{x^2}$$

$x \to +\infty$ f è int. $\forall \alpha$

$x \to 3$

$$\frac{\lg(1+(x-3)) = (x-3)^{\alpha+1}(1+o(1))}{(x-3)^{1/2}(1+o(1))}$$

$$= \frac{1}{(x-3)^{1/2-\alpha-1}}$$

$$= \frac{1}{(x-3)^{-1/2-\alpha}}$$

f è integrabile per $-\frac{1}{2}-\alpha < 1$

$-\alpha < 3/2$

$\alpha > -3/2$

$\alpha = -1$ ok

$$\int_3^{+\infty} \frac{1}{\sqrt{e^{x-3}-1}}\,dx$$

$t^2 = e^{x-3}-1 \Rightarrow e^{x-3} = t^2+1$

$x-3 = \lg(1+t^2)$

$x = 3 + \lg(1+t^2)$

$$dx = \frac{2t}{1+t^2}\,dt$$

$$\int \frac{1}{\sqrt{e^{x-3}-1}}\,dx = \int \frac{1}{\cancel{t}}\,\frac{2\cancel{t}}{1+t^2}$$

$= 2\operatorname{arctg} t + c$

$$2\operatorname{arctg}\sqrt{e^{x-3}-1}\,\Big|_3^{+\infty} = 2\cdot\frac{\pi}{2} = \pi$$

2a) Studiare, al variare di $\alpha>0$ la conv. di

$$\sum n^{2/3}\left|\operatorname{sen}\frac{1}{n}-\frac{1}{n^\alpha}\right|$$

oss. $a_n = n^{2/3}\left|\operatorname{sen}\frac{1}{n}-\frac{1}{n^\alpha}\right| = n^{2/3}\left|\frac{1}{n}-\frac{1}{6n^3}+o\left(\frac{1}{n^4}\right)-\frac{1}{n^\alpha}\right|$

se $\boxed{\alpha=1}$ $a_n = n^{2/3}\frac{1}{6n^3}(1+o(1)) = \frac{1}{n^{3-2/3=7/3}}(1+o(1))$

$\Rightarrow$ poiché $7/3>1$ per criterio confronto asintotico, la serie $\sum a_n$ converge

se $\alpha\neq 1$ $\quad a_n = n^{2/3}\left|\frac{1}{n}-\frac{1}{n^\alpha}+o\left(\frac{1}{n}\right)\right|$

se $\alpha>1 \Rightarrow a_n = O\left(\frac{1}{n^{1-2/3}}\right) = O\left(\frac{1}{n^{1/3}}\right)$

$\Rightarrow$ la serie $\sum a_n$ diverge

se $\alpha<1$ $\quad a_n = \frac{1}{n^{\alpha-2/3}}(1+o(1))$

$\Rightarrow$ la $\sum a_n$ conv. se $\alpha-\frac{2}{3}>1 \quad \alpha>1+\frac{2}{3}=5/3$

incompatibile con $\alpha<1$

Quindi in def. la serie conv. se $\alpha=1$

4b) Det. il carattere di

$$\sum \underbrace{\left(1-\frac{1}{3n}\right)^{2n^2}}_{b_n}$$

$b_n > 0$ per il criterio della radice

$$(b_n)^{1/n} = \left(1-\frac{1}{3n}\right)^{2n} = e^{2n\log\left(1-\frac{1}{3n}\right)} = e^{2n\left(-\frac{1}{3n}+o\left(\frac{1}{n}\right)\right)}$$

$$= e^{-2/3}(1+o(1))$$

osservo $e^{-2/3} < 1$

$\Rightarrow$ la serie converge