Versione D

1) Calcolare al variare di $\alpha\in\mathbb{R}$ se

$$\lim_{x\to 0^+}\frac{x(\operatorname{arctg}x)\lg(1+x^\alpha)}{1-e^{x+x^2}+x+\frac{1}{2}x\,\mathrm{sen}\,x+x^2}$$

NUM: se $\alpha>0$ $x^\alpha\to 0$ per $x\to 0^+$

$\Rightarrow$ $\lg(1+x^\alpha)=x^\alpha(1+o(1))$

Quindi $x\operatorname{arctg}x\,\lg(1+x^\alpha)=x\cdot x(1+o(1))\cdot x^\alpha=x^{\alpha+2}(1+o(1))$

oss se $\alpha=0$ $\lg(1+x^\alpha)=\lg 2$

$\Rightarrow$ NUM $\sim x^2(\lg 2+o(1))$

se $\alpha<0$ $x^\alpha\to+\infty$ per $x\to 0$

$\Rightarrow$ $\lg(1+x^\alpha)=(\lg x^\alpha)(1+o(1))=\alpha\lg x(1+o(1))$

$\Rightarrow$ NUM $=x^2\lg(x)(\alpha+o(1))$

DENOMINATORE

$1-e^{x+x^2}=\cancel{1}-\cancel{1}-(x+x^2)-\frac{1}{2}(x+x^2)^2-\frac{1}{3!}(x+x^2)^3+o(x^3)$

$=-x-x^2-\frac{1}{2}x^2-x^3-\frac{1}{6}x^3+o(x^3)$

$\frac{1}{2}x\,\mathrm{sen}\,x=\frac{1}{2}x\left(x-\frac{1}{6}x^3+o(x^4)\right)=\frac{1}{2}x^2-\frac{1}{12}x^4+o(x^5)$

Quindi DEN $=\cancel{-x}\cancel{-x^2}\cancel{+x}\cancel{+x^2}\cancel{-\frac{1}{2}x^2}-\frac{1}{6}x^3\cancel{+\frac{1}{2}x^2}-x^3+o(x^3)$

$=-\frac{7}{6}x^3(1+o(1))$

Quindi, se $\alpha>0$

$$\frac{x^{\alpha+2}(1+o(1))}{-\frac{7}{6}x^3}\to\begin{cases}0 & \text{se } \alpha+2>3 \quad \alpha>1\\ 6/7 & \text{se } \alpha+2=3 \quad \alpha=1\\ -\infty & \text{se } \alpha\in(0,1)\end{cases}$$

se $\alpha=0$ $\quad\dfrac{x^2\lg 2}{-\frac{7}{6}x^3}\to-\infty$

se $\alpha<0$ $\quad\dfrac{x^2\lg x}{-7/6\,x^3}\sim\dfrac{\lg x}{-x}\to+\infty$

1 es. versione A (cfr sol. V.D per approfondimenti)

$$\frac{e^{x+x^2}-1-x-x^2-\frac{1}{2}\operatorname{sen} x^2}{x^\alpha \operatorname{sen}\, \lg(1+x^2)}$$

$e^{x+x^2}-1 = x+x^2+\frac{1}{2}(x+x^2)^2+\frac{1}{3!}(x+x^2)^3+o(x^3)$

=(cfr. versione D) $= x+x^2+\frac{1}{2}x^2+x^3+\frac{1}{6}x^3+o(x^3)$

$-\frac{1}{2}\operatorname{sen}(x^2) = -\frac{1}{2}x^2+o(x^3)$

NUM. $= \cancel{-\frac{1}{2}x^2}+o(x^3)+\cancel{\frac{1}{2}x^2}+x^3+\frac{1}{6}x^3+o(x^3)$
$= \frac{7}{6}x^3(1+o(1))$

DEN

se $\alpha>0 \rightarrow x^{2+\alpha}(1+o(1))$ $\quad f\sim\frac{7}{6}x^{3-2-\alpha}(1+o(1))$ ($1-\alpha$)

se $\alpha=0 \rightarrow x^2(\lg 2+o(1))$ $\quad f\sim\frac{7}{6}x(1+o(1))\rightarrow 0$

se $\alpha<0 \rightarrow x^2\lg x(1+o(1))$ $\quad f\sim\frac{7}{6}\frac{x}{\lg x}\rightarrow 0$

$$\lim = \begin{cases} 0 & 1-\alpha>0\ 0<\alpha<1 \\ 7/6 & \alpha=1 \\ +\infty & \alpha>1 \\ 0 & \text{se } \alpha\le 0 \end{cases}$$

1 es V. B (cfr sol. versione D)

$$\lim_{x\to 0^+}\frac{(\cos x-1)\lg(1+x^2)}{\lg(1+x+x^2)-\frac{1}{2}x\operatorname{sen}x-x}$$

DEN:

$\lg(1+x+x^2) = x+x^2-\frac{1}{2}(x+x^2)^2+\frac{1}{3}(x+x^3)$
$+o(x^3) = x+x^2-\frac{1}{2}x^2-x^3$
$+\frac{1}{3}x^3+o(x^3)$
$= x+\frac{1}{2}x^2-\frac{2}{3}x^3+o(x^3)$

$-\frac{1}{2}x\operatorname{sen}x = -\frac{1}{2}x(x+o(x^2)) = -\frac{1}{2}x^2+o(x^3)$

DEN: $= -\frac{2}{3}x^3(1+o(1))$

NUM : $-\frac{1}{2}x^2(1+o(1))\,\lg(1+x^\alpha)$

$\lg(1+x^\alpha) = x^\alpha(1+o(1))$ se $\alpha>0$

$= \lg 2$ se $\alpha=0$

$= \lg(x^\alpha)(1+o(1)) = \alpha\lg x(1+o(1))$ se $\alpha<0$

$$\Rightarrow \frac{-\frac{1}{2}x^2}{-\frac{2}{3}x^3}\lg(1+x^\alpha) = \frac{3}{4x}\lg(1+x^\alpha)$$

$$\to \begin{cases} 0 & \text{se } \alpha>1 \\ \frac{3}{4} & \text{se } \alpha=1 \\ +\infty & \text{se } \alpha\in[0,1) \\ -\infty & \text{se } \alpha<0 \end{cases}$$

1 versione C ((cfr se. vers. D)

NUM

$\cos x - 1 - x + \lg(1+x+x^2) =$ (cfr ver. B)

$-\cancel{\frac{1}{2}x^2} + o(x^3) \cancel{-x} + \cancel{x} + \cancel{\frac{1}{2}x^2} - \frac{2}{3}x^3 + o(x^3)$

$= -\frac{2}{3}x^3(1+o(1))$

DEN $\quad x\,\mathrm{tg}x\,\lg(1+x^\alpha) = x^2\lg(1+x^\alpha)$

$$\Rightarrow \frac{-\frac{2}{3}x^3(1+o(1))}{x^2\lg(1+x^\alpha)} \simeq -\frac{2/3\,x}{\lg(1+x^\alpha)}$$

$$\to \begin{cases} -\infty & \alpha>1 \\ -\frac{2}{3} & \text{se } \alpha=1 \\ 0 & \text{se } \alpha\in[0,1) \\ 0 & \alpha<0 \end{cases}$$

Veronese D

2) Discutere, al variare di $\alpha \in \mathbb{R}$ la convergenza del seguente integrale improprio e calcolarlo, se esiste finito, per $\alpha = 0$

$$I_\alpha = \int_2^4 \frac{\sqrt{x^2-4}}{3x(x-2)^\alpha}\,dx$$

In $[2,4]$ la funzione potrebbe non essere limitata per $x \to 2^+$ (dipende da α).

Se $f_\alpha(x) = \dfrac{\sqrt{x^2-4}}{3x(x-2)^\alpha}$

per $x \to 2^+$ $\quad f_\alpha(x) = \dfrac{(x-2)^{1/2}(x+2)^{1/2}}{3x\,(x-2)^\alpha} = \dfrac{(x+2)^{1/2}}{3x\,(x-2)^{\alpha-1/2}}$

l'integrale converge se $\alpha - \frac{1}{2} < 1$ cioè $\alpha < \frac{3}{2}$

$\Rightarrow$ per $\alpha = 0$ l'integrale converge (anzi è un int. di Riemann)

$$I_0 = \int_2^4 \frac{\sqrt{x^2-4}}{3x}\,dx$$

$= \quad t = \sqrt{x^2-4} \Rightarrow x^2 - 4 = t^2 \Rightarrow x^2 = t^2+4 \Rightarrow x = \sqrt{t^2+4}$

$dx = \dfrac{1}{2\sqrt{t^2+4}}\,2t\,dt$

$x = 2 \Rightarrow t = 0$

$x = 4 \Rightarrow t = \sqrt{12} = 2\sqrt{3}$

$$\int_0^{2\sqrt{3}} \frac{t}{3\sqrt{t^2+4}}\,\frac{t}{\sqrt{t^2+4}}\,dt = \frac{1}{3}\int_0^{2\sqrt{3}} \frac{t^2+4-4}{t^2+4}\,dt$$

$$= \frac{1}{3}\,2\sqrt{3} - \frac{4}{3}\int_0^{2\sqrt{3}} \frac{1}{t^2+4}\,dt = \frac{2}{\sqrt{3}} - \frac{2}{3}\arctan\left(\frac{t}{2}\right)\Big|_0^{2\sqrt{3}}$$

$$\int \frac{1}{t^2+4}\,dt = \int \frac{1}{4\left(1+\left(\frac{t}{2}\right)^2\right)} \underset{y=\frac{t}{2}\quad dy=\frac{1}{2}dt}{=} \frac{1}{2}\arctan\left(\frac{t}{2}\right)$$

Quindi

$$I_e = \frac{2}{\sqrt{3}} - \frac{2}{3} \operatorname{arctg}(\sqrt{3}) = \frac{2}{\sqrt{3}} - \frac{2}{3}\frac{\pi}{3}$$

2° es. versione A

$$f_\alpha = \frac{\left(x-\sqrt{3/2}\right)^{1/2}\left(x+\sqrt{\tfrac{3}{2}}\right)^{1/2}}{x\left(x-\sqrt{\tfrac{3}{2}}\right)^{2\alpha}}$$

$$x \to \sqrt{\frac{3}{2}} \qquad f_\alpha \sim O\left(\frac{1}{\left(x-\sqrt{\frac{3}{2}}\right)^{2\alpha-1/2}}\right)$$

2) I_α conv. $\iff$ $2\alpha-\frac{1}{2}<1$ $\quad 2\alpha<3/2$ $\quad \alpha<\frac{3}{4}$

Il calcolo di I_0 è analogo alla versione D

2° esercizio versione B

I_α conv. $\iff$ $3\alpha-\frac{1}{2}<1$ $\qquad 3\alpha<\frac{3}{2}$ $\implies \alpha<\frac{1}{2}$

Calcolo di I_0 analogo alla versione D

2° esercizio versione C

I_α converge $\iff$ $2\alpha-\frac{1}{2}<1$ cioè $\alpha<3/4$

essendo la funzione integranda

$$O\left(\frac{1}{(x-\sqrt{2})^{2\alpha-1/2}}\right) \text{ per } x\to\sqrt{2}$$

Calcolo di I_0 analogo a versione D

versione D

3) Dopo aver dato la def di ordine di infinitesimo per una successione $\{a_n\}$

a) det. se a_n è infinitesima ed in caso, det. se esiste l'ordine di infinitesimo

$$a_n = \frac{\log(1+n^{1/5}) - \frac{1}{5}\log n}{\sqrt{n^3+3n^2}\,(\sqrt{n+7}-\sqrt{n})}$$

b) determinare, al variare di $\beta \in \mathbb{R}$ il carattere di $\sum_{n=1}^{\infty} n^\beta a_n$

Ordine di infinitesimo (def) vedere testi

a)

$$a_n = \frac{\log\left(\frac{1+n^{1/5}}{n^{1/5}}\right)}{n^{3/2}(1+o(1))\,\sqrt{n}\left(\left(1+\frac{7}{n}\right)^{1/2}-1\right)}$$

$$= \frac{\log\left(\frac{1}{n^{1/5}}+1\right)}{n^2\left(\cancel{1}+\frac{7}{2n}-\cancel{1}+o\left(\frac{1}{n}\right)\right)} = \frac{\frac{1}{n^{1/5}}(1+o(1))}{7n(1+o(1))}$$

$$= \frac{1}{n^{1+1/5}}(c+o(1))$$

$$= \frac{1}{n^{6/5}}(c+o(1))$$

$\Rightarrow$ a_n tende a zero per $n\to+\infty$ con ordine $\frac{6}{5}$

b) $\sum n^\beta a_n$

$$b_n = n^\beta a_n = \frac{n^\beta}{n^{6/5}}(c+o(1)) = \frac{1}{n^{6/5-\beta}}(c+o(1))$$

$(a_n > 0)$

$\Rightarrow \sum$ converge $\iff \frac{6}{5}-\beta>1 \;|\; \beta<\frac{1}{5}$

3° esercizio versione A

$$a_n = \frac{\lg\left(\frac{1+n^{1/6}}{n^{1/6}}\right)}{n^{3/2}(1+o(1))\, n^{1/2}\left[\left(1+\frac{7}{n}\right)^{1/2}-1\right]} \qquad \lg\left(\frac{1+n^{1/6}}{n^{1/6}}\right) = \frac{1}{n^{1/6}}(1+o(1))$$

$$\left(1+\frac{7}{n}\right)^{1/2}-1 = \cancel{1}+\frac{7}{2n}\cancel{-1}+o\left(\frac{1}{n}\right)$$

$$a_n = \frac{1}{n^{1/6+2-1}} = \frac{1}{n^{1/6+1}} = 7/6 \quad \text{ordine è } 7/6$$

b) $\sum n^\beta a_n \sim \sum \frac{1}{n^{\frac{7}{6}-\beta}}$ quindi la serie converge sse $\beta < \frac{7}{6}-1=\frac{1}{6}$

Versione B

$$a_n = \frac{\frac{1}{n^{1/3}}(1+o(1))}{n^2\sqrt{n}\left[\left(1+\frac{5}{n}\right)^{1/2}-1\right]} \sim \frac{1}{n^{1/3+1+\frac{1}{2}}} \qquad \frac{2+6+3}{6}$$

$$\left(1+\frac{5}{n}\right)^{1/2}-1 \sim \frac{5}{2n}$$

$= \frac{1}{n^{11/6}}$ ordine di infinitesimo $11/6$

$\sum n^\beta \frac{1}{n^{11/6}} \sim \sum \frac{1}{n^{\frac{11}{6}-\beta}} \Rightarrow$ la serie conv. $\Leftrightarrow \frac{11}{6}-\beta>1$

$\beta < \frac{11}{6}-1=\frac{5}{6}$

Versione C

$$a_n = \frac{\frac{1}{n^{1/4}}(1+o(1))}{n\sqrt{n}\left[\left(1+\frac{6}{n}\right)^{1/2}-1\right]} = \frac{\frac{1}{n^{1/4}}(1+o(1))}{\cancel{n}\sqrt{n}\left(\frac{6}{2\cancel{n}}(1+o(1)\right)}$$

$$= \frac{1}{n^{\frac{1}{4}+\frac{1}{2}}} = \frac{1(1+o(1))}{n^{\frac{3}{4}}}$$

ordine $= 3/4$

$$\sum n^\beta a_n \sim \sum \frac{1}{n^{3/4-\beta}}$$

converge per $\frac{3}{4}-\beta>1$

$$\beta < \frac{3}{4}-1 = -\frac{1}{4}$$

4) Data la funzione (Vernaleo)

$$f(x) = -\frac{1}{4} \lg\left(\frac{x^2}{|x-3|}\right) + x + 4$$

studiarla e disegnarne un grafico qualitativo

dom f $\quad x \neq 3 \qquad x \neq 0$

$\lim_{x\to 3} f(x) = -\infty \qquad x=3$ asintoto verticale dx e sin.

$\lim_{x\to 0} f(x) = +\infty \qquad x=0$ asintoto verticale

$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x - \frac{1}{4}\lg(x)(1+o(1)) = +\infty$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x - \lg(-x) = -\infty$ non ci sono asint. obliqui né a $+\infty$ né a $-\infty$

$$f(x) = \begin{cases} x+4 - \frac{1}{4}\lg\left(\frac{x^2}{x-3}\right) & x>3 \\ x+4 - \frac{1}{4}\lg\left(\frac{x^2}{3-x}\right) & x<3 \quad x \neq 0 \end{cases}$$

nel suo dominio f è derivabile e

$$f'(x) = \begin{cases} 1 - \frac{1}{4} \frac{\cancel{x-3}}{x^{\cancel{2}}} \frac{2x(x-3) - x^2}{(x-3)^{\cancel{2}}} \quad = x^2 - 6x = x(x-6) & x>3 \\ 1 - \frac{1}{4} \frac{\cancel{3-x}}{x^{\cancel{2}}} \frac{2x(3-x)+x^2}{(3-x)^{\cancel{2}}} = 6x - x^2 = x(6-x) & x<3 \end{cases}$$

$$= \begin{cases} 1 - \frac{1}{4} \frac{x-6}{x(x-3)} & x>3 \\ 1 - \frac{1}{4} \frac{6-x}{x(3-x)} & x<3 \quad x \neq 0 \end{cases}$$

$$= \begin{cases} \frac{4x(x-3) - x + 6}{4x(x-3)} & x>3 \\ \frac{4x(3-x) - 6 + x}{4x(3-x)} & x<3 \quad x \neq 0 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{4x^2-13x+6}{4x(x-3)} & x>3 \\ \dfrac{-4x^2+13x-6}{4x(3-x)} & x<3 \quad x \neq 0 \end{cases}$$

$$4x^2-13x+6=0$$

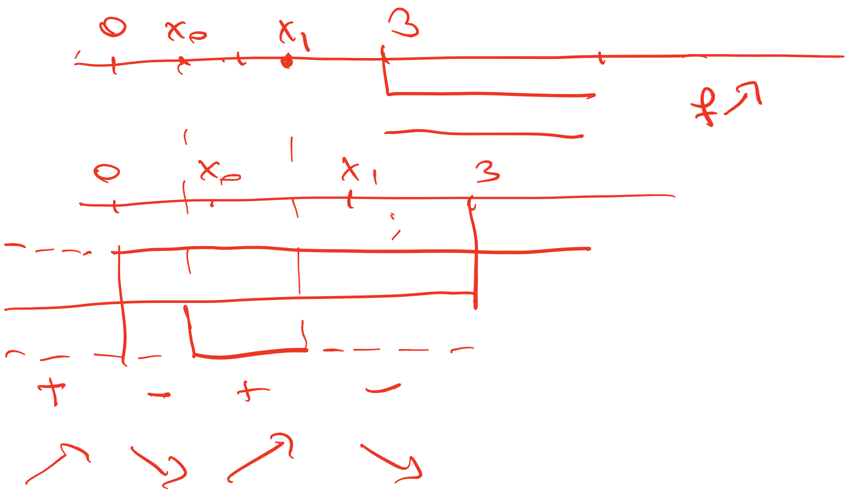
$$x = \frac{13 \pm \sqrt{169-96}}{8} = \frac{13 \pm \sqrt{73}}{8}$$

$$x_0 = \frac{13-\sqrt{73}}{8} \qquad x_1 = \frac{13+\sqrt{73}}{8} < 3$$

$x_0 > 0$ $\quad 13 > \sqrt{73}$ sì

$13+\sqrt{73} < 24$

$\sqrt{73} < 11$ sì



$x = x_0$ punto di minimo rel.

$x = x_1$ punto di max. rel.

$$f'(x) = \begin{cases} \dfrac{4x^2-13x+6}{4x(x-3)} & x>3 \\ \dfrac{-4x^2+13x-6}{4x(3-x)} & x<3 \quad x \neq 0 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{4x^2-13x+6}{4x(x-3)} & x>3 \\ \dfrac{-4x^2+13x-6}{4x(3-x)} & x<3 \quad x\neq 0 \end{cases}$$

$$f' = \begin{cases} \dfrac{4x^2-13x+6}{4x(x-3)} & x>3 \\ + \dfrac{(4x^2-13x+6)}{4x(x-3)} & x<3 \end{cases}$$

quindi f' ha la stessa espressione per $x>3$ e $x<3$

$4x^2-12x$

$$f'' = \frac{(8x-13)(4x^2-12x) - (8x-12)(4x^2-13x+6)}{16x^2(x-3)^2}$$

segno f'' =

$$\text{segno}\left(\cancel{32x^3} - 96x^2 - 52x^2 + \cancel{156x} - \left(\cancel{32x^3} - 104x^2 + 48x - 48x^2 + \cancel{156x} - 72\right)\right)$$

$$(-96 - 52 + 104 + 48)x^2 - 48x + 72$$

$$\text{segno}(4x^2 - 48x + 72) = \text{segno}(x^2 - 12x + 18)$$

ma $x^2 - 12x + 18 = 0$

$$x_{2,3} = 6 \pm \sqrt{36-18} = 6 \pm \sqrt{18}$$

x_3 flesso $6+\sqrt{18} > 3$

$x_2 = 6-\sqrt{18} \in (x_0, x_1)$

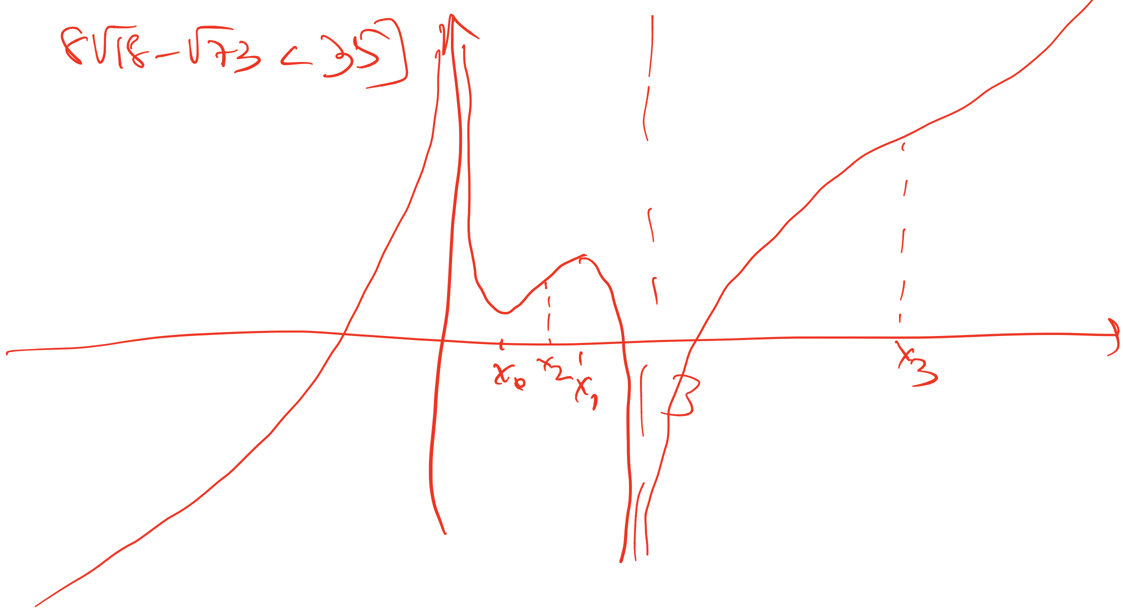
f conv. se $x > x_3$, $x < x_2$

concava altr.

$$0 < \frac{13-\sqrt{73}}{8} < 6-\sqrt{18} < \frac{13+\sqrt{73}}{8} < 3$$

$[13-\sqrt{73} < 48 - 8\sqrt{18}$

$8\sqrt{18} - \sqrt{73} < 35]$



Versione C (cfr vers D per approf.)

ES 4

$$f(x) = -\frac{1}{6}\lg\left(\frac{x^2}{|x+2|}\right) - x + 3$$

dom $x \neq 0 \quad x \neq -2$

$x \to 0 \qquad f \to +\infty \qquad$ as. vert.

$x \to -2 \qquad f \to -\infty \qquad$ as. vert.

$x \to +\infty \qquad f \simeq -x - \frac{1}{6}\lg x \to -\infty$

$x \to -\infty \qquad f \simeq -x - \frac{1}{6}\lg(-x) \to +\infty$

$$f = \begin{cases} -x + 3 - \frac{1}{6}\lg\left(\frac{x^2}{x+2}\right) & x > -2 \quad x \neq 0 \\ -x + 3 - \frac{1}{6}\lg\left(\frac{x^2}{-x-2}\right) & x < -2 \end{cases}$$

$$f' = \begin{cases} -1 - \frac{1}{6}\,\frac{\cancel{x+2}}{x^{\cancel{2}}}\,\frac{2\cancel{x}(x+2) - x^{\cancel{2}}}{(x+2)^{\cancel{2}}} & = 2x - x + 4 = x + 4 \\ -1 + \frac{1}{6}\,\frac{(\cancel{-x-2})}{x^{\cancel{2}}}\,\frac{2\cancel{x}(-x-2) + x^{\cancel{2}}}{(x+2)^{\cancel{2}}} & = -2x - 4 + x = -x - 4 \end{cases}$$

$$= \begin{cases} \frac{-6x^2 - 12x - x - 4}{6x(x+2)} = \frac{-6x^2 - 13x - 4}{6x^2+12x} = -\frac{(6x^2+13x+4)}{6x^2+12x} & x > -2 \\ \frac{-6x^2 - 12x - x - 4}{6x(x+2)} = \frac{-(6x^2+13x+4)}{6x(x+2)} & x < -2 \end{cases}$$

$6x(x+2) = 6x^2 + 12x$

$$6x^2 + 13x + 4 = 0$$

$$x = \frac{-13 \pm \sqrt{169 - 96}}{12} = \frac{-13 \pm \sqrt{73}}{12}$$

$$f' = \frac{-(6x^2+13x+4)}{6x(x+2)}$$

$$f'' = \frac{-(12x+13)(6x^2+12x) + (6x^2+13x+4)(12x+12)}{36x^2(x+2)^2}$$

sgn f'' =

sgn $\left[\left(-(12\cdot 6) + \cancel{(12\cdot 6)}\right)x^3 - 144x^2 - 78x^2 - \cancel{13\cdot 12x} \right.$

$\left. + 72x^2 + \cancel{13\cdot 12x} + \underbrace{(13\cdot 12)}_{156}x^2 + 48x + 48 \right]$

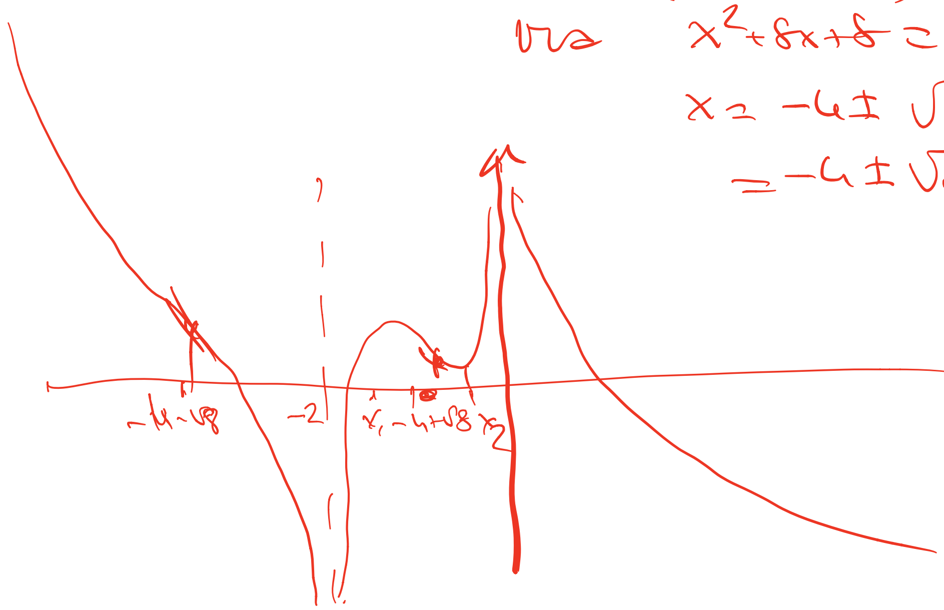
$= \text{sgn}\left[6x^2 + 48x + 48\right]$

$6(x^2 + 8x + 8)$

vra $x^2 + 8x + 8 = 0$

$x = -4 \pm \sqrt{16-8}$

$= -4 \pm \sqrt{8}$



Versione B (cfr Vers D per approf)

$$f(x) = -\frac{1}{4}\lg\left(\frac{x^2}{|x-2|}\right) + x + 2$$

$x \neq 2$ $\quad x \neq 0$

$x \to 2 \quad f \to -\infty$

$x \to 0 \quad f \to +\infty$

$x \to +\infty \quad f \to +\infty$

$x \to -\infty \quad f \to -\infty$

$$f = \begin{cases} x+2-\frac{1}{4}\lg\left(\frac{x^2}{x-2}\right) & x>2 \\ x+2-\frac{1}{4}\lg\left(\frac{x^2}{2-x}\right) & x<2 \end{cases}$$

$$f' = \begin{cases} 1-\frac{1}{4}\,\frac{\cancel{x-2}}{x^{\cancel{2}}}\,\frac{2\cancel{x}(x-2)-x^{\cancel{2}}}{(x-2)^{\cancel{2}}} = x-4 \quad = \frac{4x^2-8x-x+4}{4x(x-2)} \\ 1-\frac{1}{4}\,\frac{\cancel{2-x}}{x^{\cancel{2}}}\,\frac{2\cancel{x}(2-x)+x^{\cancel{2}}}{(2-x)^{\cancel{2}}} = 4-x \quad = \frac{8x-4x^2-4+x}{4x(2-x)} \quad x<2 \end{cases}$$

$$f' = \begin{cases} \frac{4x^2-9x+4}{4x(x-2)} & x>2 \\ \frac{4x^2-9x+4}{4x(x-2)} & x<2 \quad x \neq 0 \end{cases}$$

$4x^2-9x+4=0$

$$x = \frac{9 \pm \sqrt{81-64}}{8} = \frac{9\pm\sqrt{17}}{8}$$

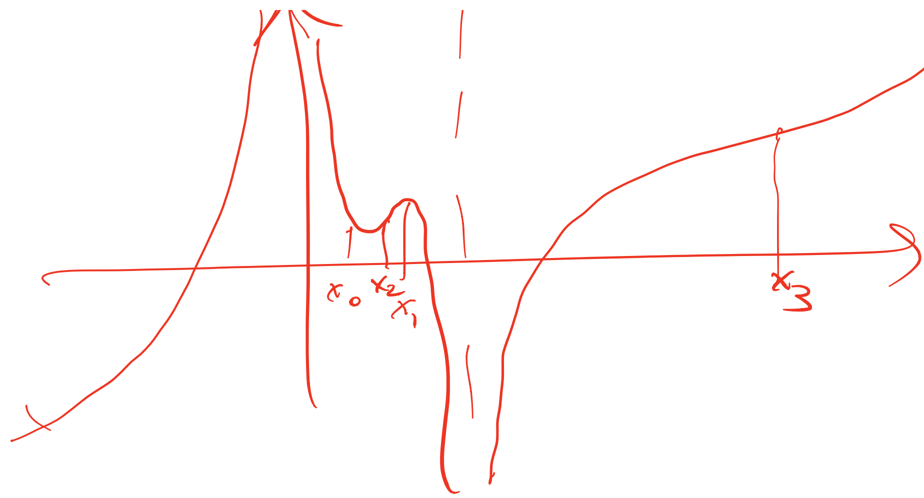
81
64
17

$$f'' = \frac{(8x-9)(4x^2-8x)-(8x-8)(4x^2-9x+4)}{16x^2(x-2)^2}$$

$\mathrm{sgn} = \mathrm{sgn}\,(-64x^2-36x^2+\cancel{72x}+72x^2-32x+32x^2-\cancel{72x}+32$

$4x^2-32x+32$

$4(x^2-8x+8)=0 \qquad x_{1,2} = 4\pm\sqrt{16-8} = 4\pm\sqrt{8}$



Versione A (grafico versione D)

$$f = -\frac{1}{3}\lg\left(\frac{x^2}{|x+3|}\right) - x + 1$$

dom $x \neq 0$ $\quad x \neq -3$

$x \to -\infty \quad f \to +\infty$
$x \to +\infty \quad f \to -\infty$ } no asintoti obliqui

$x \to 0 \quad f \to +\infty$
$x \to -3 \quad f \to -\infty$ } asintoti verticali

$$f = \begin{cases} 1 - x - \frac{1}{3}\lg\left(\frac{x^2}{x+3}\right) & x > -3 \quad x \neq 0 \\ 1 - x - \frac{1}{3}\lg\left(\frac{x^2}{-3-x}\right) & x < -3 \end{cases}$$

$$f' = \begin{cases} -1 - \frac{1}{3}\frac{\cancel{(x+3)}}{x^2}\frac{2\cancel{x}(x+3) - x^2}{(x+3)^2} = x+6 \\ -1 \mp \frac{1}{3}\frac{\cancel{(+3+x)}}{x^2}\frac{2\cancel{x}(-3-x) + x^2}{(3+x)^2} = -6 - x = +(6+x) \end{cases}$$

$$f' = \frac{-9x - 3x^2 - 6 - x}{3x(3+x)} = \frac{-(3x^2 + 10x + 6)}{3x(3+x)}$$

$$3x^2+10x+6=0$$

$$x=\frac{-5\pm\sqrt{25-18}}{3}=\frac{-5\pm\sqrt{7}}{3}=x_{0,1}$$

$$f''=\frac{-(6x+10)(9x+3x^2)+(3x^2+10x+6)(9+6x)}{9x^2(3+x)^2}$$

$$\text{sign } f''=\text{sign}\big[-54x^2-\cancel{18x^3}-\cancel{90x}-30x^2+27x^2+\cancel{18x^3}+\cancel{90x}+60x^2+54+36x\big]=$$

$$=\text{sign}(3x^2+36x+54)$$

ons $3x^2+36x+54=0$

$$x^2+12x+18=0$$

$$x=-6\pm\sqrt{36-18}=-6\pm\sqrt{18}=x_{2,3}$$