January 28th, 2016

Bachelor's Degree in ENGINEERING SCIENCES

Mathematical Analysis I

Prof. F. Ciolli – Prof. T. D'Aprile

Exercise 1. Sketch the graph of the following function

$$f(x) = |2x - 1|\, e^{\frac{1}{x-3}}$$

specifying: domain, possible asymptotes, monotonicity, continuity, local and global maxima or minima, corner points (if any) and non-derivability points.

Exercise 2. For any $\alpha \in \mathbb{R}$, the following function is given

$$f(x) = \begin{cases} |\alpha| \arctan\left(\dfrac{1}{x}\right), & x > 0 \\ (\alpha+1)x^2 - 2x + 1\,, & x \leq 0\,. \end{cases}$$

1. Find the values of α (if any) such that f result to be continuous or differentiable or globally invertible on its domain;
2. Fix $|\alpha| = 1$ and write the equation of the tangent line to the graph of the (local) inverse function f^{-1} of f, in the point $\left(\frac{\pi}{4}, f^{-1}\left(\frac{\pi}{4}\right)\right)$.

Exercise 3. Consider the integral

$$I_\alpha = \int_2^{+\infty} f_\alpha(x)\, dx\,, \quad \text{for} \quad f_\alpha(x) = \frac{\ln^{3\alpha-5}\left(\sqrt{x}+1\right)}{(x+3)^\alpha} \quad \text{and} \quad \alpha \in \mathbb{R}\,.$$

1. Discuss the integrability of I_α in the improper sense, for any $\alpha \in \mathbb{R}$;
2. Fix $\alpha = 2$ and calculate (if any) the value of I_2;
3. Fix $\alpha = 2$ and calculate the order of infinite/infinitesimal for $x \to 0^+$ of the function

$$f_2(x) - \frac{1}{9}\left(\sin(\sqrt{x})\cos(\sqrt{x}) - \arcsin\left(\frac{x}{2}\right)\right)\,.$$

Exercise 4. Find the local minimum and local maximum points (if any) of the following function:

$$f(x,y) = e^{x^2} + y^4 - 1, \quad (x,y) \in \mathbb{R}^2.$$

Exercise 5. Find all complex numbers z which satisfy the following equation and express them in the Cartesian form $a + \mathrm{i}b$:

$$z^6 + \mathrm{i}z^3 = 0.$$

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February 19th, 2016

Bachelor's Degree in ENGINEERING SCIENCES

Mathematical Analysis I

Prof. F. Ciolli – Prof. T. D'Aprile

Exercise 1. Sketch the graph of the following function

$$f(x) = \sqrt[3]{|x+3|(x^2-2)}\,,$$

specifying: domain, possible asymptotes, monotonicity, continuity, local and global maxima or minima, and non-derivability points (specifying the type, if any). Moreover, write the equation of the tangent line to the graph of f in the point $(0, f(0))$.

Exercise 2. Considered the function

$$f(t) = \frac{\arctan(\log t)}{t\,(\log t - 1)^2}\,,$$

discuss the integrability and calculate, if possible, the integrals:

$$\text{2.a)} \quad \int_{e^2}^{e^3} f(t)dt\,; \qquad\qquad \text{2.b)} \quad \int_{e}^{+\infty} f(t)dt\,.$$

Exercise 3. Discuss the absolute and simple convergence of the following series:

$$\text{3.a)} \quad \sum_{n=1}^{\infty} \left(\frac{n\tan(1+\frac{1}{n^2})}{n+1}\right)^n\,; \qquad\qquad \text{3.b)} \quad \sum_{n=0}^{\infty} \left(\frac{x-1}{x+2}\right)^n \log\left(\frac{n+1}{n+2}\right)\,, \text{ for } x \in \mathbb{R}\,.$$

Exercise 4. Find the solution of the following Cauchy problem:

$$\begin{cases} y'' + 2y' + 2y = 5xe^x \\ y(0) = 0 \\ y'(0) = 1 \end{cases}.$$

Exercise 5. Compute the following limit:

$$\lim_{n\to+\infty} (n^4 + \log n)\Big(\log\left(1+\frac{1}{n^2}\right) - \frac{1}{n}\sin\left(\frac{1}{n}\right)\Big).$$

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June 22nd, 2016

Bachelor's Degree in ENGINEERING SCIENCES

Mathematical Analysis I

Prof. F. Ciolli – Prof. T. D'Aprile

Exercise 1. Sketch the graph of the following function

$$f(x) = x - \frac{3x^2+4}{|x|-1},$$

specifying: domain, possible asymptotes, monotonicity, continuity, local and global maxima or minima, and non-derivability points (specifying the type, if any). Moreover, write the equation of the tangent line to the graph of f in the point $(-2, f(-2))$.

Exercise 2. For $\alpha \in \mathbb{R}^+ \cup \{0\}$, study the integrability of

$$\int_0^{+\infty} \frac{\log^2\left(\arctan(t^\alpha)+1\right)}{1+t^{2\alpha}}\, dt\,.$$

Then calculate, if any, the value of the integral for $\alpha = 1$.

Exercise 3. Compute the following limit:

$$\lim_{x\to+\infty} \frac{\left(e^{\frac{x^2}{1-x}} - e^{-(x+1)}\right)\left(\sin\left(x^{-\frac{1}{3}}\right) - \frac{1}{6}\sin\left(\frac{1}{x}\right) - x^{-\frac{1}{3}}\right)}{e^{-(x+2)}\arctan(3+x)\left(1-\cos\left(x^{-\frac{4}{3}}\right)\right)}.$$

Exercise 4. Find all complex numbers z which satisfy the following equation and express them in the Cartesian form $a + \mathrm{i}b$:

$$(z+\mathrm{i})^3 = \frac{1-\mathrm{i}}{1+\mathrm{i}}.$$

Exercise 5. Find the local minimum, local maximum and saddle points (if any) of the following function:

$$f(x, y) = x\log(1 + x^2 + y^2).$$

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July 14th, 2016

Bachelor's Degree in ENGINEERING SCIENCES

Mathematical Analysis I

Prof. F. Ciolli – Prof. T. D'Aprile

Exercise 1. Sketch the graph of the following function

$$f(x) = \ln(1+ \mid 3e^x - 2e^{2x} - 1|)\,,$$

specifying: domain, possible asymptotes, monotonicity, continuity, local and global maxima or minima, and non-derivability points (specifying the type, if any). Moreover, write the equation of the tangent line to the graph of f in the point $(1, f(1))$.

Exercise 2. Discuss the absolute and simple convergence of the following series, for $\alpha \in \mathbb{R}$:

$$2.a) \quad \sum_{k=1}^{\infty}(-1)^k \left(\ln\left(1-\frac{1}{k}\right)+\frac{1}{k}\right); \quad 2.b) \quad \sum_{k=1}^{\infty} e^{\frac{(\alpha^2-1)k+\sin\left(\frac{1}{k}\right)-\alpha\frac{1}{k}}{k}} - 1\,.$$

Exercise 3. Study the boundedness of the following numerical set

$$A = \left\{\frac{1}{n-\ln n},\ n \in \mathbb{N}\setminus\{0\}\right\} \bigcup \left\{\frac{x+2}{x+1},\ x \in \mathbb{R}^+ \setminus \{0\}\right\}\,,$$

expressing $\sup A$ and $\inf A$, $\max A$ and $\min A$ if any. Then determine the set of its interior points $\mathring{A}$, its boundary ∂A and the set of its accumulation points A'.

Exercise 4. Find the solution of the following Cauchy problem:

$$\begin{cases} y' - \dfrac{2x-1}{x^3-x}y = 0 \\ y(2) = 1 \end{cases}.$$

Exercise 5. Compute the following limit:

$$\lim_{n\to+\infty} \frac{4n! + n^n + 5e^n n^2}{(n+1)^n}.$$

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September 5th, 2016

Bachelor's Degree in ENGINEERING SCIENCES

Mathematical Analysis I

Prof. F. Ciolli – Prof. T. D'Aprile

Exercise 1. Sketch the graph of the following function

$$f(x) = \frac{x^3}{|x| - 2} - x^2\,,$$

specifying: domain, possible asymptotes, monotonicity, continuity, local and global maxima or minima, and non-derivability points (specifying the type, if any). Moreover, write the equation of the tangent line to the graph of f in the point $(1, f(1))$.

Exercise 2. Study the simple convergence of the following series and integral for $\alpha \in \mathbb{R}$. Then calculate, if it exists, the value of the integral in **2.b)** for $\alpha = 1$.

2.a) $$\sum_{k=1}^{+\infty} \left(\frac{1}{k^\alpha} - \sin\frac{1}{k} \right)\,,$$ **2.b)** $$\int_1^{+\infty} \frac{\arctan(x^\alpha + 2)}{x^{2\alpha}}\, dx\,.$$

Exercise 3. Study the boundedness of the numerical set $A = I \cup B$, where

$$I = \left\{ \frac{2}{x+1}\,,\ \text{for } x \in (-1, 1) \right\} \quad \text{and} \quad B = \left\{ \frac{1}{2} - \frac{n}{n^2+1}\,,\ \text{for } n \in \mathbb{N} \cup \{0\} \right\},$$

expressing $\sup A$ / $\inf A$ and $\max A$ / $\min A$, if any. Then determine the set of its interior points $\mathring{A}$, of its boundary ∂A and the set of its accumulation points A'.

Exercise 4. Compute the following limit:

$$\lim_{n\to+\infty} n!\Big(\exp\left(\frac{1}{4^n}\right) - 1 - \frac{1}{2^n} \sin\frac{1}{2^n} \Big)\,.$$

Exercise 5. Find the local minimum, local maximum and saddle points (if any) of the following function:

$$f(x, y) = e^{x^2}(x^3 + 3y^2)\,.$$

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September 19th, 2016

Bachelor's Degree in ENGINEERING SCIENCES

Mathematical Analysis I

Prof. F. Ciolli – Prof. T. D'Aprile

Exercise 1. Sketch the graph of the following function

$$f(x) = \ln\left(\frac{1+ \mid e^{2x} - e^x|}{e^x + 2}\right),$$

specifying: domain, possible asymptotes, monotonicity, continuity, local and global maxima or minima, and non-derivability points (specifying the type, if any). Moreover, write the equation of the tangent line to the graph of f in the point $(1, f(1))$.

Exercise 2. Compute the following limit

$$\lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right) - e^{\left(\frac{1}{6x^2}+\frac{1}{e^x}\right)} + \cos\left(\sqrt{\frac{2}{x}}\right)}{\ln\left(2+\tan\left(\frac{1}{x}\right)\right) - \ln\left(2+\arctan\left(\frac{1}{x}\right)\right)}.$$

Exercise 3. Study the convergence of the following series

3.a) $$\sum_{k=1}^{+\infty} \frac{k^2 \sin\left(\frac{1}{k^{\frac{1}{\alpha}}}\right) + e^{(\alpha-1)k}}{k+1}, \ \alpha \in \mathbb{R} \setminus \{0\} ;$$ **3.b)** $$\sum_{n=2}^{+\infty} \frac{n!(n+1)!}{(n^2-n)!}.$$

Exercise 4. Find the solution of the following Cauchy problem:

$$\begin{cases} y' - 3x^2 y = 3x^5 \\ y(0) = 1 \end{cases}$$

Exercise 5. Find all complex numbers z which satisfy the following equation and express them in the Cartesian form $a + \mathrm{i}b$:

$$(z+\sqrt{2})^2 + \frac{4}{1+\mathrm{i}\sqrt{3}} = 0.$$

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