Numerical study (Valk et al. 2009) – Analytical study (Lemaître et al. 2009)

Dynamics of space debris – Very high A/m – Geostationary orbit
Gravitational coefficient C22 – Direct solar radiation pressure

Numerical study (Valk et al. 2009) – Analytical study (Lemaître et al. 2009)

Addition of tumbling effect

Geostationary resonant angle $\Omega$ and Sun longitude $\lambda$

**Context of this study**

Dynamics of space debris – Very high A/m – Geostationary orbit
Gravitational coefficient C22 – Direct solar radiation pressure

**Numerical study:**
- Very high A/m
- Geostationary orbit
- Gravitational coefficient C22
- Direct solar radiation pressure

**Analytical study:**
- Numerical integration of motion equation of the dynamics of GEO space debris (A/m = 10 m²/kg)
- Including the main forces:
  - Numerical integration of motion equation of the dynamics of GEO
  - Gravitational coefficients
  - Direct solar radiation pressure

**References**

- Laskar, J., 1990
- Chao C.C., 2009
- Valk et al., 2009
- Delsate N*, Lemaître A. and Carletti T.

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**Numerical integration of motion equation of the dynamics of GEO space debris (A/m = 10 m²/kg) including the main forces**

**Frequency analysis of the signal: diffusion indicator – EOD**

- Frequency analysis: approximation of a signal $f(t)$
  - From the numerical knowledge of $f(t)\approx \sum_{n} a_n e^{i \omega_n t}$
  - Over a finite time span [T1, T2]

**Time diffusion of the main frequencies with respect to the time.**

- Computation of the main frequency ($\nu$) of the signal:
  - $\nu_1$: first value on a time span [0, 55]
  - $\nu_2$: second value on a time span [55, 110]

**Estimate of the diffusion rate**

- Computation of a numerical estimate of derivative of frequency with respect to the time: EOD.

- EOD = $\log (\nu_2^2 - \nu_1^2) / (\nu_2 - \nu_1)$

The smaller its value is, the smaller the diffusion is.

**Addition of a periodic variation of A/m (attitude motion or tumbling) with a high averaged area-to-mass ratio (A/m = 10 m²/kg)**

- Choose a simplified sinusoidal variation [Chao, 2009]
- $A = A_0 (1 - k \sin (G \cdot t))$
  
  Where ($A_0$/m) is the constant initial value of A/m
  - $k$ is the amplitude of the sine term. Here equal 0.35
  - $G$ is the spin rate of the object.

- Map with second derivative of the main frequency, equivalent to EOD.

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**Numerical study - additional resonances**

**Numerical Study:**

- The chaotic motion of the solar system. A numerical estimate of the size of the chaotic zones, Icarus 88, 286-291.
- Chao C.C., 2009 Poster at the Fifth European conference on Space Debris, Darmstadt.

**References**

- Laskar, J., 1990
- Chao C.C., 2009
- Valk et al., 2009
- Delsate N*, Lemaître A. and Carletti T.

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**Numerical Study: impact of the tumbling effect onto the dynamics of GEO space debris**

- Tumbling with period equal to 1 day
- More stable and more additional resonances

**Associated papers**

- Global dynamics of high area-to-mass ratios GEO space debris by means of the MEGNO indicator (Valk et al., Advances in Space Research 43, 2009, p 1509–1526)