Random homogenization.

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First rigorous results on homogenization of elliptic and stationary parabolic operators with random coefficients were obtained by S.Kozlov and then independently by G.Papanicolaou and S.Varadhan in the late seventies. It was shown that a family of divergence form uniformly elliptic operators

$$A^\varepsilon = \mathrm{div}\left(a\Big(\frac{x}{\varepsilon},\omega\Big)\nabla\right)$$

whose coefficients are the realizations of a statistically homogeneous ergodic matrix-field $a_{ij}(x,\omega)$, admits almost surely homogenization, as $\varepsilon \downarrow 0$, and that, moreover, the limit operator, also called effective diffusion operator, has nonrandom constant coefficients.

One of the important particular examples of random media is high contrast random checker-board structure. This model exhibits a number of interesting asymptotics, its study relies essentially on a number of results obtained in the percolation theory.

In contrast with the periodic framework, the generalisation of basic homogenization results to random elliptic operators with lower order terms encounters serious difficulties. There is a number of counterexamples showing that in general the homogenization theorem may fail to hold for such operators. For instance, in the model introduced by Ya.Sinai, one can observe the subdiffusive behaviour of solutions, while in some random stratified models the superdiffusive behaviour occurs.

Recent years a certain progress was achieved in studying the problems involving nonstationary parabolic operators whose coefficients are random in time and periodic in spatial variables. In particular, it has been shown that for operators with divergence form elliptic part the usual homogenization theorem holds. In the presence of lower order terms we have, in general, weaker averaging result. Namely, the law of solutions of the original problem converges, in a proper functional space, to a solution of limit stochastic partial differential equation.

We will first focus on the proof of homogenization theorem for divergence form elliptic operators. After that, we will discuss several particular models and examples, and then consider some nonstationary evolution models.

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