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Dynamic scaling in natural swarms

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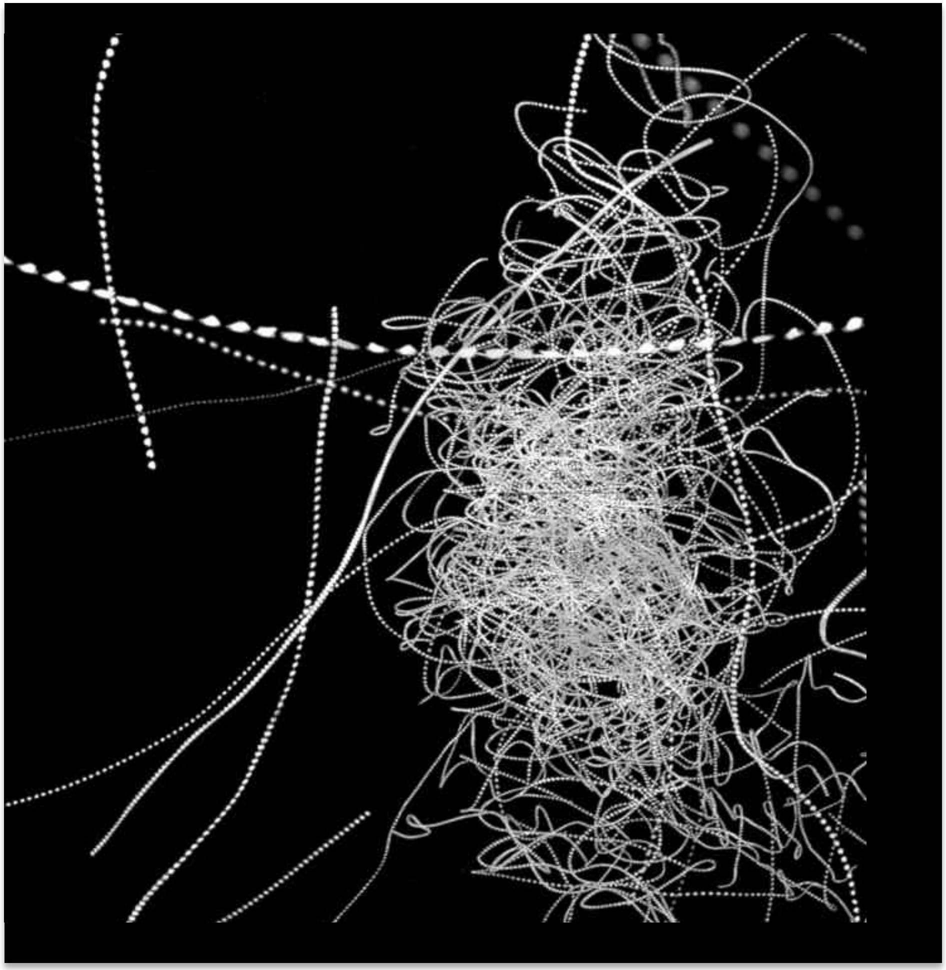
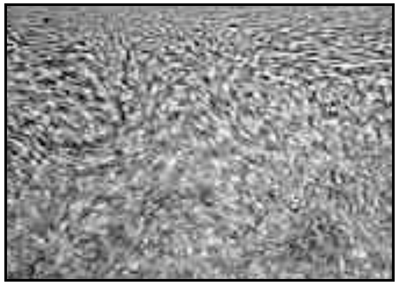


CNR

Institute
for Complex Systems

Accademia Nazionale dei Lincei, Rome 2017

Collective behaviour in biological systems

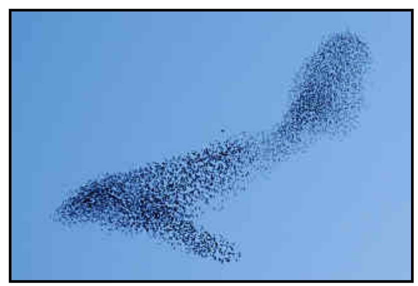


bacteria

cells

insects

fish

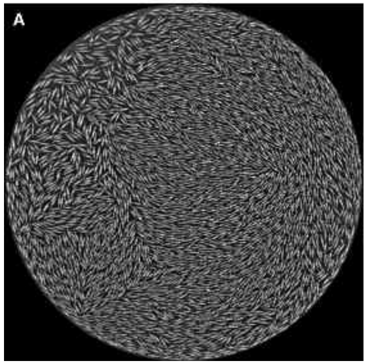


birds

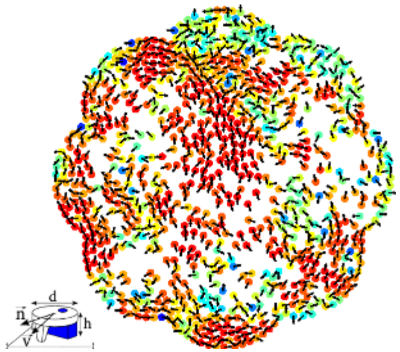
mammals

self-organized collective behaviour

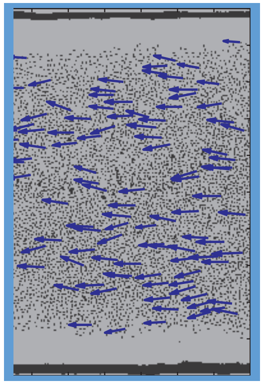
Analogy with statistical physics and active non-living matter



Narayan et al, Science 2007



Deseigne et al, PRL 2010



Bricard et al, Nature 2013

Hope: details do not matter – few features determine the universality class
simple models capture the large scale behavior of active matter

$$\begin{cases} \vec{v}_i(t+1) = \vec{v}_i(t) + J\sum_{k\in i}\vec{v}_k(t) + \vec{\xi}_i & |\vec{v}_i| = v_0 \\ \vec{r}_i(t+1) = \vec{r}_i(t) + \vec{v}_i(t+1) & \end{cases}$$

self-propelled particles
Vicsek model

Are these hopes/assumptions justified for animal groups and complex biological systems?

Collective behaviour in animal groups

movie by C. Carere - Starflag

movie by S. Melillo, SWARM

Flocks

Global order
Scale free correlations - Collective turns

Pnas 105 (2008), Pnas 107 (2010), Pnas 109 (2012)
Nature Phys 10 (2014), Jstat 2015, Nature Phys 12 (2016)
PRL 118 (2017)

Swarms

No global order
Correlations – quasi critical behavior

Plos Comp. Biol 10 (2014) , PRL 113 (2014)
Nature Phys 13 (2017)

Can we define classes of behavior ?

scaling in critical phenomena

static scaling

correlation function in momentum space:

control parameters

$$C_s(k) = C_s(k; \alpha_1, \ldots, \alpha_n)$$

correlation length:

$$\xi = \xi(\alpha_1, \ldots, \alpha_n)$$

static scaling hypothesis:

$$\text{for } \xi \gg a$$ microscopic scale

$$C_s(k; \alpha_1, \ldots, \alpha_n) = \frac{1}{k^{2-\eta}} f(k\xi)$$

anomalous dimension

dynamic scaling

dynamic scaling hypothesis:

$$C(k,t) = C_s(k)\, g(t/\tau_k,\, k\xi)$$

$$\tau_k = \frac{1}{k^z}\, h(k\xi)$$

dynamic critical exponent

dynamical renormalization group idea:

$$x \to x/b \quad \Rightarrow \quad t \to t/b^z$$

consequences of dynamic scaling

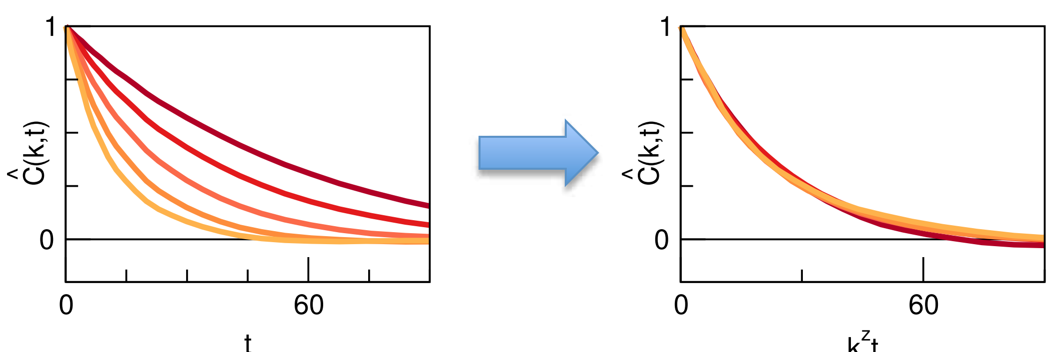
collapse

$$C(k,t) = C_s(k)\, g(t/\tau_k,\, k\xi)$$

$$\tau_k = \frac{1}{k^z}\, h(k\xi)$$

$$\boxed{\text{if } k\xi = \text{constant}}$$

$$C(k,t)/C_s(k) \equiv \hat{C}(k,t) = g(k^z t)$$



just one function

critical slowing down

$$\tau_k = \frac{1}{k^z}\, h(k\xi)$$

$$\boxed{\text{if } k\xi = \text{constant}}$$

$$\tau_k \sim k^{-z} \sim \xi^z$$

systems strongly correlated in space are also strongly correlated in time

THE JOURNAL OF CHEMICAL PHYSICS VOLUME 43, NUMBER 11 1 DECEMBER 1965

Equation of State in the Neighborhood of the Critical Point

B. WIDOM
Department of Chemistry, Cornell University, Ithaca, New York
(Received 15 July 1965)

Physics Vol. 2, No. 6, pp. 263-272, 1966. Physics Publishing Co. Printed in Great Britain.

SCALING LAWS FOR ISING MODELS NEAR T_c*

LEO P. KADANOFF †
Department of Physics, University of Illinois
Urbana, Illinois

PHYSICAL REVIEW VOLUME 177, NUMBER 2 10 JANUARY 1969

Scaling Laws for Dynamic Critical Phenomena

B. I. HALPERIN AND P. C. HOHENBERG
Bell Telephone Laboratories, Murray Hill, New Jersey 07974
(Received 13 August 1968)

VOLUME 28, NUMBER 4 PHYSICAL REVIEW LETTERS 24 JANUARY 1972

Critical Exponents in 3.99 Dimensions*

Kenneth G. Wilson and Michael E. Fisher
Laboratory of Nuclear Studies and Baker Laboratory, Cornell University, Ithaca, New York 14850
(Received 11 October 1971)

scaling

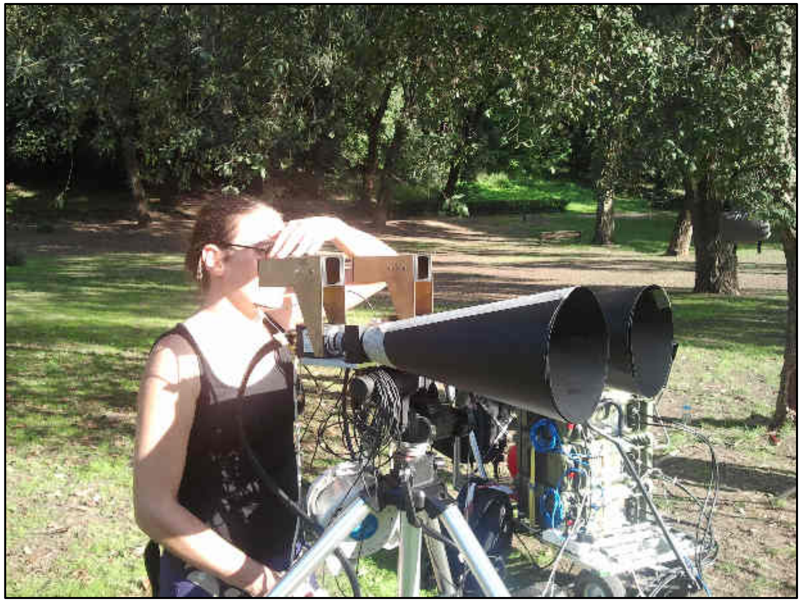
renormalization group

universality

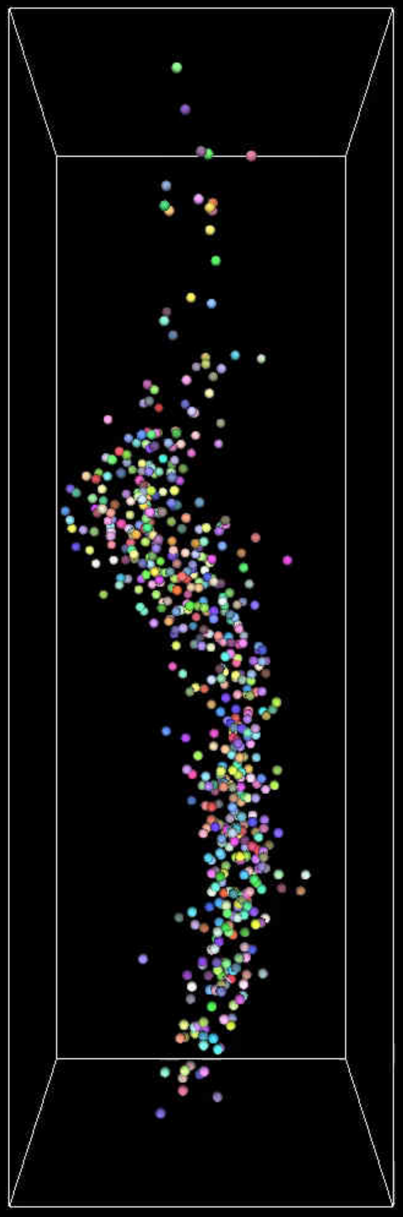
universality in biological systems?

start from scaling

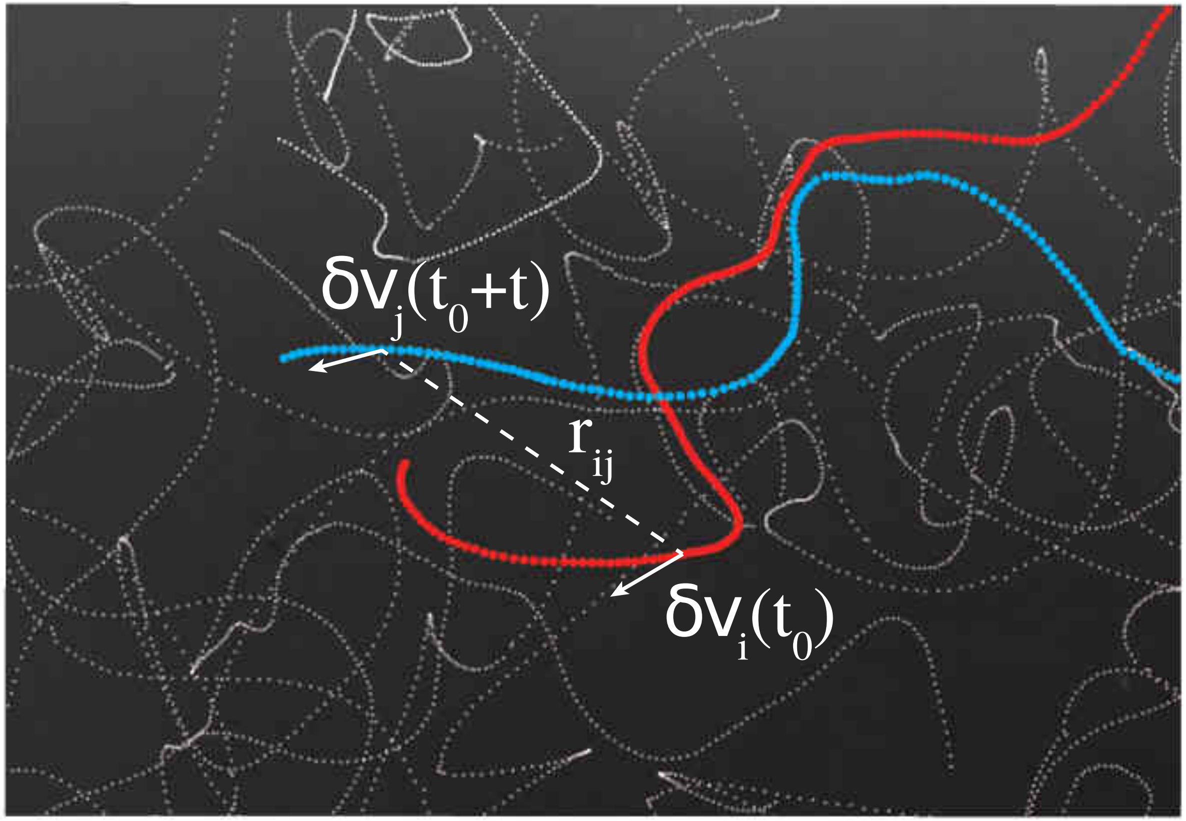
experiments




1 mm
this is our `spin’



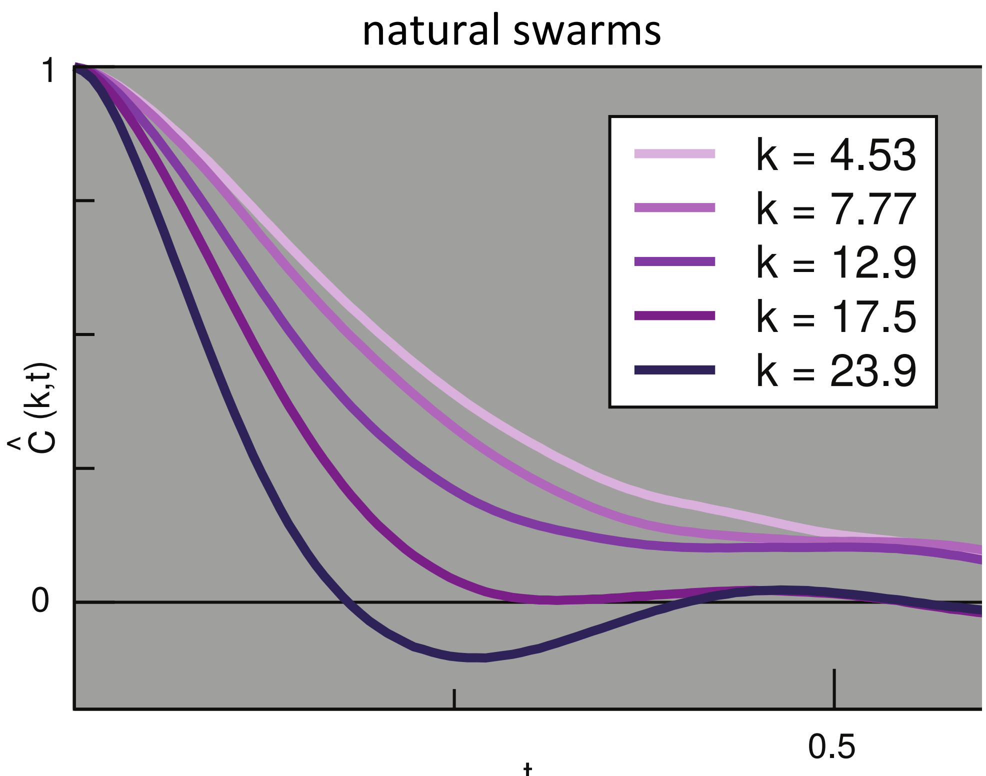
velocity correlation function in real space and time



$$C(r,t) = \left\langle \frac{\sum_{i,j}^{N} \delta\hat{\mathbf{v}}_i(t_0)\cdot\delta\hat{\mathbf{v}}_j(t_0+t)\,\delta[r - r_{ij}(t_0,t)]}{\sum_{i,j}^{N} \delta[r - r_{ij}(t_0,t)]} \right\rangle_{t_0}$$

dynamic correlations in *k*-space

$$C(k,t) = \left\langle \frac{1}{N} \sum_{i,j}^{N} \frac{\sin[k\, r_{ij}(t_0,t)]}{k\, r_{ij}(t_0,t)} \delta\hat{\mathbf{v}}_i(t_0) \cdot \delta\hat{\mathbf{v}}_j(t_0+t) \right\rangle$$



$$\hat{C}(k,t) \equiv C(k,t)/C(k,t=0)$$

dynamic scaling hypothesis

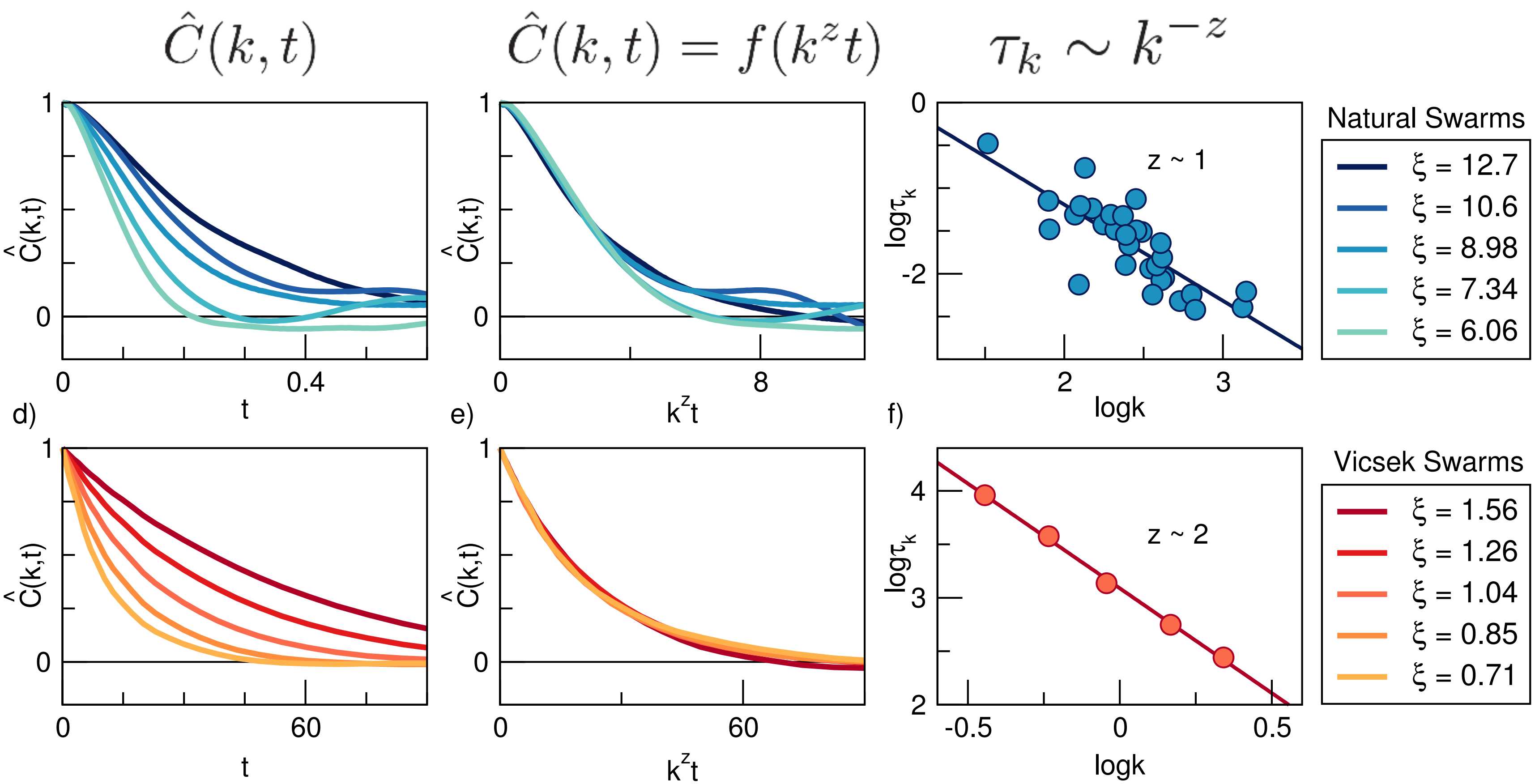
$$\hat{C}(k,t) = f(t/\tau_k; k\xi),$$
$$\tau_k = k^{-z} g(k\xi),$$

$$\boxed{\text{if } k\xi = \text{constant}}$$

$$\hat{C}(k,t) = f(k^z t)$$
$$\tau_k \sim k^{-z}$$

$$\boxed{\text{we will fix } k = 1/\xi}$$

dynamic scaling holds and a new exponent emerges



- natural swarms: $z = 1.0 \pm 0.2$
- Vicsek swarms: $z = 2.0 \pm 0.03$

how anomalous is the exponent $z = 1$?

equilibrium models:

- Heisenberg/Ising model (Model A): $z = 2$
- Non-dissipative antiferromagnet (linear spin-wave) (Model G): $z = 1.5$
- Quantum ferromagnet (Model J): $z = 2.5$

active matter models:

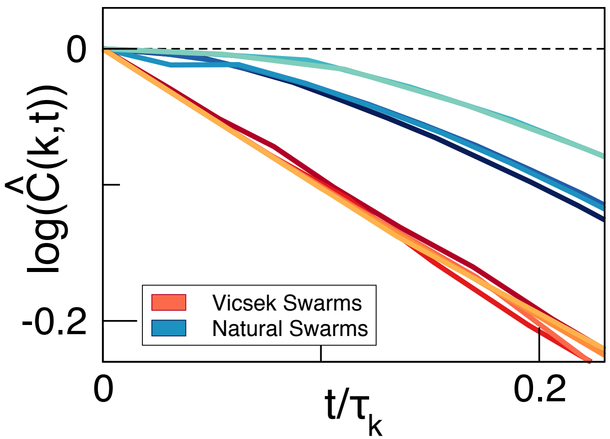
- Vicsek model, ordered phase (flocks): $z = 1.6$
- Vicsek model, disordered phase (swarms): $z = 2$

such a small value of z seems to require a non-trivial renormalization structure scaling

renormalization group

universality

non-dissipative relaxation

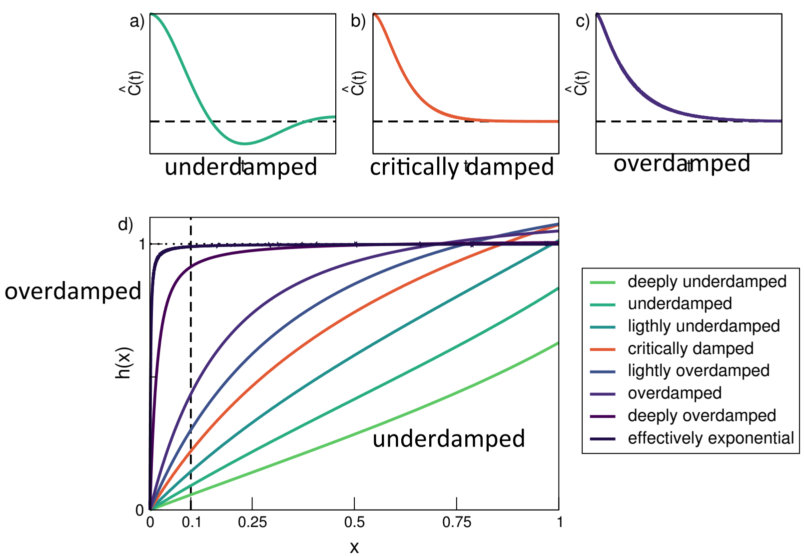


$$h(x) \equiv -\frac{1}{x}\log \hat{C}(x) \quad , \quad x \equiv t/\tau_k$$

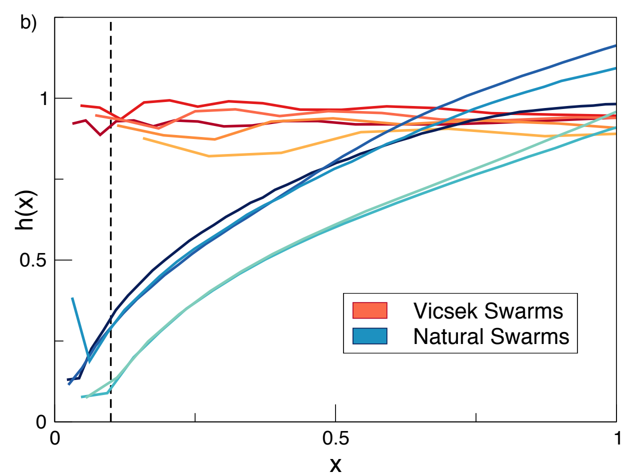
$\lim_{x\to 0} h(x) = 1$ exponential relaxation - dissipative

$\lim_{x\to 0} h(x) = 0$ non-exponential relaxation – non-dissipative

stochastic harmonic oscillator



swarms

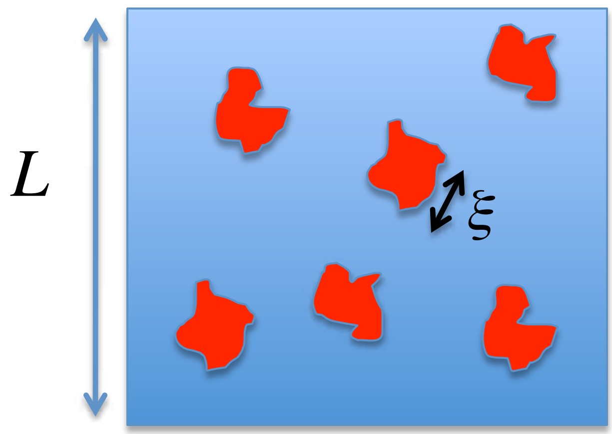


second-order inertial dynamics ?

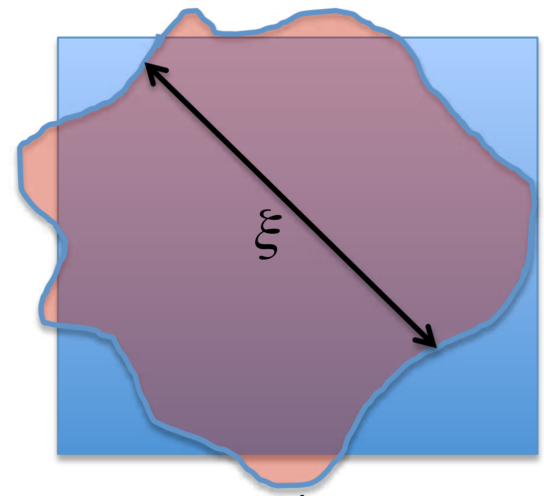
the system has no long-range order

why don’t we observe a purely dissipative regime?

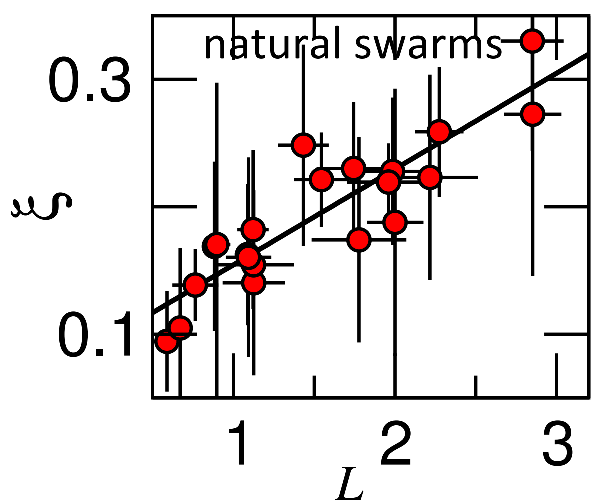
hydrodynamic vs critical regime



hydrodynamic regime



critical regime
hydrodynamics breaks down



$$\xi \sim L$$

swarms are always in the critical regime

near-critical censorship of hydrodynamics

conclusions

dynamic scaling holds in natural swarms

anomalous critical exponent and non-dissipative relaxation

near-critical censorship of hydrodynamics

arXiv:1611.08201
Nature Physics, 06/2017

COBBS Group – Collective Behaviour in Biological Systems

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