XIV International Congress on Mathematical Physics
Lisbon, July 28 - August 2, 2003

On the Work of Huzihiro Araki

Laudatio delivered by R. Longo
on the occasion of the Poincaré prize to H. Araki

Mathematical Physics: whatever definition you take, I presume it will fall between the following two

$$\text{Minimal definition:} \quad M \cap \Phi$$
$$\text{Maximal definition:} \quad M \cup \Phi$$

Huzihiro Araki is mathematical physicist in the most distinguished sense. He is both a mathematician and a physicist.

The work of Araki has been mostly concerned with the Operator Algebra approach to Quantum Field Theory, Statistical Mechanics and the internal structure of von Neumann algebras. In all these subjects his contributions are fundamental.

It is not possible to give here a survey of his very extensive work. We shall content ourselves by mentioning a few highlights in his impressive production.

Mathematical work

Araki-Woods classification of ITPFI factors. Let $\mathcal{H}$ be a Hilbert space. A von Neumann algebra $\mathcal{M}$ on $\mathcal{H}$ is a unital *-subalgebra of $B(\mathcal{H})$ closed in the operator-weak topology, equivalently $\mathcal{M} = \mathcal{M}''$ (von Neumann density theorem) where S' denotes the commutant of a set $S \subset \mathcal{B}(\mathcal{H})$. A *factor* is a von Neumann algebra with trivial center: $\mathcal{M}' \cap \mathcal{M} = \mathbb{C}$.

By using Powers factors, Araki and Woods classified ITPFI factors on a separable Hilbert space, i.e. factors of the form

$$\mathcal{M} = \bigotimes_{i=1}^{\infty} {}^{\{\Omega_i\}} \mathcal{M}_i$$

where $\mathcal{M}_i \simeq B(\mathcal{H}_i)$. This complete classification for a class of factors goes beyond Murray-von Neumann work and was a precursor of A. Connes' classification of injective factors.

Positive cones, noncommutative L^p-spaces. If $\mathcal{M}$ is an abelian von Neumann algebra, then $\mathcal{M} \simeq L^\infty(X,\mu)$ acting on $\mathcal{H} \equiv L^2(X,\mu)$. The positive cone $L^2(X,\mu)_+$ and the L^p-spaces can then be defined.

Based on Tomita-Takesaki modular theory, Araki (and independently both Connes and Haagerup) defined in particular a natural positive cone associated to a general von Neumann algebra $\mathcal{M}$ with a cyclic and separating vector Ω, modular operator and conjugation Δ, J,

$$L^2(\mathcal{M},\Omega)_+ = \overline{\Delta^{\frac{1}{2}}\mathcal{M}_+\Omega} = \{aJaJ\Omega, a \in \mathcal{M}\}^-$$

The "magic" properties of $L^2(\mathcal{M},\Omega)_+$ are at the foundation of many subsequent discoveries in Operator Algebras.

Various approaches to noncommutative L^p -spaces have been pursued. Araki's one based on positive cones is one of the most natural.

Quantum Field Theory

In Haag's approach to Local Quantum Physics, a physical system is described by a net

$$\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$$

of von Neumann algebras $\mathcal{A}(\mathcal{O})$ on a Hilbert space $\mathcal{H}$, associated with regions $\mathcal{O}$ of the spacetime $\mathbb{R}^{d+1}$. Locality (= Einstein causality) is expressed by commutativity at spacelike distance: $\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}')'$.

The basic model is associated with a free Bose field, where $\mathcal{H} = \mathrm{Fock}(\mathcal{H}_0)$. Araki made a first pioneering work: he constructed a *lattice* of von Neumann algebras

$$\mathcal{K} \subset \mathcal{H}_0 \text{ real Hilbert subspace} \mapsto \mathcal{A}(\mathcal{K})$$

In particular Haag duality holds

$$\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}')$$

a property at the basis of much structure analysis in QFT, e.g. in the Doplicher-Haag-Roberts study of superselection sectors.

Another important work of Araki in QFT concerns his contribution to the Haag-Ruelle *scattering theory*.

Quantum Statistical Mechanics

Relative entropy. In a finite quantum system, the relative entropy between states ω_1, ω_2 (= positive linear normalized functionals) is given by

$$S(\omega_1|\omega_2) = -\mathrm{Tr}(\rho_1 \log \rho_1 - \rho_1 \log \rho_2)$$

ρ_i density matrices of ω_i.

At *infinite volume*, where the observables generate a von Neumann algebra $\mathcal{M}$, Araki considered the *relative modular operator* $\Delta_{\Omega_2,\Omega_1}$

associated with the unique vector representatives of ω_i in $\Omega_i \in L^2(\mathcal{M})_+$ and defined the fundamental notion of relative entropy

$$S(\omega_1|\omega_2) = (\Omega_1, \log \Delta_{\Omega_2,\Omega_1}\Omega_1)$$

which shares all the natural expected properties: $S(\omega_1|\omega_2) \leq 0$, etc ...

Araki's relative entropy is a basic tool in many papers both in Math and Physics. For example it entered in my own work on the quantization of the incremental entropy values (proportional to $\log n$) for a quantum black hole.

KMS and variational principle. Haag, Hugenholtz and Winnink proposed the following characterization of thermal equilibrium states. If the time evolution is described by a one-parameter automorphism group α of the observable C^*-algebra $\mathfrak{A}$, then a state ω of $\mathfrak{A}$ is KMS at inverse temperature $\beta > 0$ if for all $a, b \in \mathfrak{A}$

$$\omega(ba) = \underset{t \to i\beta}{\text{anal.cont.}}\, \omega(\alpha_t(a)b)$$

Araki showed that (on lattice systems)

$$KMS \Leftrightarrow \text{Variational principle}$$

providing a sound physical justification for the KMS condition, by now widely accepted.

Chemical potential. The intrinsic notion of chemical potential appeared in the work of Araki, Haag, Kastler and Takesaki. If the time evolution is described by a one-parameter automorphism group of the observable C^*-algebra $\mathfrak{A}$, and $\mathfrak{A} = \mathfrak{F}^G$ is the fixed-point subalgebra of the field algebra $\mathfrak{F}$ w.r.t. a compact gauge group G then, under suitable extremality/ergodicity assumptions, the extensions to $\mathfrak{F}$ of a β-KMS state ω_β are KMS and labeled by a parameter (one for each charge) μ, the chemical potential

$$\omega_\beta \to \tilde{\omega}_{\beta,\mu}$$

This paper is an emblematic combination of Math power and Physical insight.

An incredible organizer

I hope the mentioned scientific contributions have given you a glimpse into Araki's research activity.

The Mathematical Physics community owes a lot to Araki, not only for his scientific contributions. Araki is a famous organizer. For example he has been:

- founder of Reviews in Mathematical Physics,

- one of the editors of Communications in Mathematical Physics,

- one of the representatives at the International Mathematical Union,

- the main and essentially the only organizer of the International Congress of Mathematicians held in Kyoto in 1990,

- president of the International Association of Mathematical Physics.

The Japanese school in Operator Algebras

If today there is a flourishing school in Operator Algebras in Japan, most is due to Huzihiro Araki. His choice to live in Japan, his scientific influence, his "unlimited energy" and capability in the organization aspects have produced generations of new scientists that make the Japanese school among the top most important worldwide.

Conclusion

The work of Araki continues to be very influential. I would add that his papers are completely rigorous, providing a solid reference for future work.

Let me conclude with the hope that the Poincaré prize to Araki will point to the new generation of researchers the way Mathematical Physics may achieve the difficult task of providing completely rigorous and high quality results of interest both to Physics and Mathematics.

Congratulations,
Professor Araki!

荒木先生
おめでとうございます！

Felicitações,
Professor Araki!