



Cluster expansion in statistical mechanics and its connection with the Lovász Local Lemma in combinatorics

Corso di Dottorato tenuto dal Prof.: **Aldo Procacci**

Periodo: Inizio Aprile - fine a Maggio con 2 incontri (4 ore) a settimana

Programma del corso:

Part 1. Continuous particles in the Grand Canonical Ensemble interacting via a pair potential

1. Conditions on the pair potential: stability and regularity
2. The infinite volume limit. Existence (the case of the finite range pair potential)
3. Properties of the pressure. Continuity.
4. The Mayer series
5. The combinatorial problem
6. The Penrose tree graph identity: partition schemes.
7. Analyticity at low density/high temperature
 - a) The hard sphere gas (via the original Penrose partition scheme)
 - b) gas of particles interacting via a stable and regular pair potential (via the Kruskal algorithm partition scheme)

Part 2. Discrete systems

1. The abstract polymer gas
2. Convergence of the cluster expansion
3. Convergence criteria: Kotecký-Preiss; Dobrushin; Fernández-Procacci.
4. Elementary examples.
5. Gas of non-overlapping subsets
6. Applications: spin systems at high temperature
7. Ising model at low temperature.
8. Antiferromagnetic Potts model at zero temperature on a graph G (complex zeros of the chromatic polynomial of G).

Part 3. The Connection with the probabilistic method in combinatorics

1. A powerful tool in combinatorics: The Lovász Local Lemma
2. Shearer criterion.
3. Scott-Sokal formulation of the Shearer Criterion via the abstract polymer gas.
4. The cluster expansion Local lemma
5. Example: colorings of a graph.
6. The Moser-Tardos algorithmic version of the Lovász Local Lemma
7. Entropy-compression method.

References

1. <http://150.164.25.15/~aldo/apostila-ps-pdf.pdf>
2. <https://www.dropbox.com/s/w8535w5boelio18/APS%20versus%20LLL.pdf?dl=0>