Valerio Lucarini

“Elements of the Mathematics of Climate Variability and Climate Change”.

The course consists of 4 classes of 2 hours each during the days:

- November 16th, 10:30 - 12:30
- November 18th, 15:00 – 17:00
- November 23rd, 10:30 – 12:30
- Novembre 25th, 15:00 – 17:00

and it will take place remotely on Teams. Anyone interested can contact Prof. Piermarco Cannarsa at cannarsa@mat.uniroma2.it.

Below the abstract and the content of each class.

Abstract: “In 2013, the mathematical community, through the Mathematics for the Planet Earth (MPE2013) international initiative, coalesced around the idea of creating a common effort towards identifying the most interesting mathematical questions coming from the investigation of the Planet Earth and towards developing sophisticated tools for modelling, analysing, and predicting the behaviour of the Planet Earth. Hence, the effort was directed at the same time in the direction of developing new ideas in pure and applied mathematics, in mathematical modelling, in theoretical physics, in geosciences, and in data science. A new generation of interdisciplinary young scientists has been trained along these lines, and one can expect that this will lead to major impacts in mathematical, computational, and natural sciences in the year to come. This short course will provide an overview of some fascinating developments at the interface between mathematics, physics and geosciences. The lectures are not intended to provide an exhaustive presentation of the topics, but rather aim at presenting some concepts that can be instrumental for defining or refining research ideas. The mathematics will be presented in a rather heuristic, rather than rigorous, way

Review papers:
M. Ghil, V. Lucarini, The Physics of Climate Variability and Climate, Rev. Modern Physics, 92, 035002 (2020)
V. M. Galfi, V. Lucarini, F. Ragone, J. Wouters, Applications of Large Deviation Theory in Climate Science and Geophysical Fluid Dynamics, Rivista del Nuovo Cimento 44, 291–363 (2021)”

Lecture 1 - 2h - Climate Variability and Climate Change. We will introduce some basic phonemenology of the climate system and discuss how it is studied, through observations, modelling, and theory. We will then discuss, using the point of view of statistical mechanics, how it is possible to formulate a theory of climate change based on the use of response operators. We will discuss Ruelle's response theory and its applications, present examples of use of the response theory for performing climate projections in a hierarchy of climate models. We will also discuss in an informal way the functional analytic viewpoint based on the transfer operator.

V. Lucarini, Response Operators for Markov Processes in a Finite State Space: Radius of Convergence and Link to the Response Theory for Axiom A Systems, J. Stat. Phys. 162, 312 (2015)
A. Gritsun, V. Lucarini, Fluctuations, Response, and Resonances in a Simple Atmospheric Model, Physica D 349, 62 (2017)
V. Lucarini, F. Lunkeit, F. Ragone, Predicting Climate Change Using Response Theory: Global Averages and Spatial Patterns, J. Stat. Phys. 166, 1036 (2017)
V. Lucarini, Revising and Extending the Linear Response Theory for Statistical Mechanical Systems: Evaluating Observables as Predictors and Predictands, J. Stat. Phys. 173, 1698–1721 (2018)
V. Lembo, V. Lucarini, F. Ragone, Beyond Forcing Scenarios: Predicting Climate Change through Response Operators in a Coupled General Circulation Model, Scientific Reports 10, 8668 (2020)

Lecture 2- 2h - Parametrization of unresolved processes. The climate system has variability on a vast range of temporal and spatial scale. The absence of a clear time-scale separation between different dynamical processes require the need to go beyond averaging or homogenisation theory. We will discuss how modellers deal with the problem of coarse-graining in climate modelling and how this can be interpreted using the formalism of the Mori-Zwanzig operator. We will also present the data-driven viewpoint.

J. Wouters, V. Lucarini, Multi-level dynamical systems: Connecting the Ruelle response theory and the Mori-Zwanzig approach. J Stat Phys, 151, 850 (2013)
G. Vissio, and V. Lucarini, A proof of concept for scaleƸadaptive parametrizations: the case of the Lorenz '96 model, Q. J. Royal Met. Soci. 144, 63-75 (2017)
M. Santos-Gutierrez, V. Lucarini, M. Chekroun, M. Ghil, Reduced-Order Models for Coupled Dynamical Systems: Koopman Operator and Data-driven Methods, Chaos 31, 053116 (2021)

Lecture 3 - 2h - Multistability - Critical Transitions are a ubiquitous feature of the climate system and occur in the cases where the response operators diverge. We will explore the relationship between the occurrence of critical transitions and the presence of a spectral gap in the transfer operator. We will then explore the properties of multistable dynamical systems and the features of noise-induced transitions between the competing metastable states.

V. Lucarini, T. Bodai, Edge States in the Climate System: Exploring Global Instabilities and Critical Transitions, Nonlinearity 30, R32 (2017)
A. Tantet, V. Lucarini, H. A. Dijkstra, F. Lunkeit, Crisis of the chaotic attractor of a climate model: a transfer operator approach, Nonlinearity 31 2221 (2018)
V. Lucarini, T. Bodai, Global Stability Properties of the Climate: Melancholia States, Invariant Measures, and Phase Transitions, Nonlinearity 33 R59 (2020)
G. Margazoglou, T. Grafke, A. Laio, V. Lucarini, Dynamical landscape and multistability of a climate model, Proc. R. Soc. A.477 20210019 (2021)

Lecture 4 - 2h - Extreme Events in the Climate System. We will first introduce the relevance of extreme events in the climate system and emphasize some of the challenges associated with climate change. We will then discuss how extreme value theory applied to specific observables of chaotic dynamical systems can help understanding the fundamental properties of the system under investigation, and how such ideas can be used to shed light on some predictability properties of the climate system. We will then investigate how large deviation theory provides a powerful framework for understanding persistent extreme events and for better characterising their dynamical properties.

V. Lucarini, D. Faranda, A. Freitas, J. Freitas, M. Holland, T. Kuna, M. Nicol, M. Todd, S. Vaienti, Extremes and Recurrence in Dynamical Systems, Wiley, 2016
V. M. Galfi, T. Bódai, V. Lucarini, Convergence of Extreme Value Statistics in a Two-Layer Quasi-Geostrophic Atmospheric Model, Complexity, doi:10.1155/2017/5340858 (2017)
V. M. Galfi, V. Lucarini, Fingerprinting Heatwaves and Cold Spells and Assessing Their Response to Climate Change using Large Deviation Theory, Physical Review Letters 127, 058701 (2021) [Editor's suggestion; featured in Physics]