Doctoral thesis: Call for candidates

June 16th 2017

Title. Probabilistic numerical methods for non-linear PDEs and applications to energy management.

Key words. Energy; Numerical Methods; Management; Hamilton-Jacobi-Bellman; Non-conservative PDEs; BSDEs.

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Locations:
ENSTA ParisTech, Université Paris-Saclay,
Ecole doctorale de mathématiques Hadamard (EDMH).

Università Milano-Bicocca,
Dottorato in Matematica Milano-Bicocca, Pavia INdAM).

The student is supposed to spend at least one year in both institutions.

Profile: Basic knowledge required in Stochastic analysis, Probabilistic numerical methods. Computational skills will be highly appreciated.

Applications including CV plus a short motivation letter can be addressed by e-mail to all the addresses below:

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General context

With the emergence of renewable energies (as wind or solar generation), local generation systems are rapidly multiplying integrating renewables, batteries or more conventional plants (such as gas turbines or hydro plants). The impact of random factors (such as demand, energy prices, wind, luminosity etc.) on the management of such local generation systems are significant. Hence, an important issue is to be able to manage efficiently such microgrids in presence of uncertainties. Mathematically, the related optimization problem can be stated in terms of a stochastic control problem which can be reduced to a nonlinear Partial Differential Equation (PDE), known as Hamilton-Jacobi-Bellman (HJB) equation. One celebrated approach is based on backward stochastic differential equations (BSDEs). The thesis will focus on recent forward numerical schemes based on generalized Fokker-Planck representations for nonlinear PDEs in high space dimension. In the specific case of mass conservative PDEs, it is well-known that the solution can be probabilistically represented as the marginal densities of a Markov diffusion nonlinear in the sense of Mckean. Then one can design forward interacting particle schemes to approximate numerically the PDEs solution. The research will elaborate some extensions of this kind of representation and interacting particle scheme associated to a large class of PDEs including the case when they are non-conservative, non integrable with various kind of nonlinearities. The comparison with BSDEs approach will also be a significant task to perform.

Methods

The research related to this project will follow four main axes.

1. Theoretical developments through the statement of mathematical theorems or propositions.
2. Algorithmic developments with mathematical analysis of speed of convergence and numerical complexity.
3. Computer simulations related to the algorithms: numerical experiences on stylized examples will help to validate the practical interest.
4. Comparison with the representations of semilinear PDEs via BSDEs (Backward Stochastic Differential Equations), classical or of second-order type.

The first step for the student will consist in getting familiar with the existing literature by the advisor team, basically [3, 5, 4, 2] and more generally the quoted articles. Later, the project will consist in the elaboration of three scientific articles to be published in recognized scientific journals. The related topics will be the following.

1. Extension of the approach proposed by [4] for semilinear PDEs where the underlying process is a classical diffusion Y to the case where Y is a stochastic process, non-linear in the McKean sense.

2. Extension of the approach proposed by [3, 5] to the case of PDEs admitting solutions which are non-positive and non-integrable.

3. Formalization and mathematical analysis of a PDE representation, specifically suited to stochastic control problems making use of time reversal techniques.

References

[1] P. Del Moral. *Mean field simulation for Monte Carlo integration*, volume 126 of *Monographs on Statistics and Applied Probability.* CRC Press, Boca Raton, FL, 2013.

[2] M. Fuhrman and G. Tessitore. HJB equations through Backward Stochastic Differential Equations. In *Stochastic Optimal Control in Infinite Dimensions: Dynamic Programming and HJB Equations. To appear.* Chapter 6.

[3] A. Le Cavil, N. Oudjane, and F. Russo. Probabilistic representation of a class of non conservative nonlinear partial differential equations. *To appear: Latin American Journal of Probability and Mathematical Statistics (ALEA). Preprint HAL. https://hal.archives-ouvertes.fr/hal-01241701*, 2015.

[4] A. Le Cavil, N. Oudjane, and F. Russo. Forward Feynman-Kac type representation for semilinear nonconservative partial differential equations. *arXiv preprint arXiv:1608.04871*, 2016.

[5] A. Le Cavil, N. Oudjane, and F. Russo. Particle system algorithm and chaos propagation related to a non-conservative McKean type stochastic differential equations. *Stochastics and Partial Differential Equations: Analysis and Computation.*, pages 1–37, 2016.