Ginzburg-Landau energy with weight and pinning phenomena

Rejeb Hadiji

UPEC : University Paris Est Créteil
hadiji@u-pec.fr

Abstract

Let G be a smooth bounded domain in $\mathbb{R}^2$. We consider the functional

$$E_\varepsilon\left(u\right) = \frac{1}{2}\int_G f(x,u)\left|\nabla u\right|^2 + \frac{1}{4\varepsilon^2}\int_G J\left(1-\left|u\right|^2\right)$$

on the set $H^1_g(G,\mathbf{C}) = \left\{u \in H^1(G,\mathbf{C});\ u = g \text{ on } \partial G\right\}$ where g is a given boundary data with degree $d \geq 0$. This functional is related to the Ginzburg-Landau energy in supraconductivity. From the physical point of view we can consider the problem as a "model problem". The presence of a non constant weight is motivated by the problem of pinning vortices.

In this mini course, we will study the behaviour of minimizers u_ε of E_ε and we will estimate the energy $E_\varepsilon(u_\varepsilon)$. More precisely, we start by recall some prelimenary results. After, we will speak about the fundamental works of Bethuel-Brezis and Hélein in the cass where the weight is a constant and the potential is standard. Then, we will consider different types of weights and of potentials. We especially study the cases where the weight depends only on x, and the case where the weight $f(x,u) = p_0 + t\left|x\right|^k\left|u\right|^l$. We will especially also study the problem with a general convex potential.

Introduction for the theory of Ginzburg-Landau

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The matter of the course is give some fondamental tools and to study the problem of Ginzburg-Landau.

The first couse 24 May 2016 : Some tools, the degree theory.

Abstract of the first course : 3h. We give the definition of the degree and some other tools which we need in the Ginzburg-Landau problem.

The second course 26 May 2016 : A Ginzburg-Landau problem

Abstract of the second course : 2 or 3h. We study the problem of Ginzburg-Landau problem in some particular situation.