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Title: *"Metric Spaces of Keplerian Orbits"*
Abstract. Different questions in Celestial Mechanics lead us to study the structure of the space H of Keplerian orbits and several its subspaces H_s. First attempt to bring metrics in one of H_s , say H_1 (H without rectilinear, circular, and coplanar orbits passing in the opposite directions), was made by Southworth and Hawkins in 1963. Results were rather useful for the astronomical practice, but incorrect from the theoretical point of view. The corresponding "distance" D (named "criterion" by the authors) did not satisfy the triangle axiom. Hence the pair (H_1, D) represents a pseudo-metric space only. In 1970 Moser (and almost simultaneously Stiefel and Scheifele) proved a brilliant theorem showing that the existence of singularities in any system of elements for elliptic orbits is a direct consequence of the topology of the space H_2 of elliptic Keplerian orbits with fixed negative energy. This space has a topological structure of the product of 2-dimensional spheres. Later different authors made several modifications of D, but they all turned out to be pseudo-metric. In 2004 Kholshevnikov and Vassiliev introduced an irreproachable metric in the space H_3 of elliptic orbits. But it had a serious drawback: parabolic and hyperbolic orbits were excluded. So it was impossible to use it when comparing orbits of comets and meteoroid streams. In 2010 Maruskin proposed a Riemannian metric in H_3 regarding it as a 5-dimensional surface embedded in R^6. It suffered the same drawback. In 2008 Kholshevnikov proposed a metric in the space H_4 of all Keplerian orbits, and another one in the space H_5 of non-rectilinear Keplerian orbits. In 2015 Kholshevnikov introduced a more friendly metric in H_5, and pseudo-metrics in three factor-spaces of H_5 , where we neglect nodes, pericenters, or both nodes and pericenters. In 2017 Milanov proved that these pseudo-metrics satisfy all axioms of metric spaces. Here we describe properties of these metrics.