Analisi Matematica I
Preliminari

Esercizio 1. Determinare estremo superiore e inferiore dei seguenti insiemi

(1) $\{x \in \mathbb{R} : x^2 - 10x + 16 \leq 0\}$

(2) $\{x < -2 : x^2 - x - 6 \geq 0\}$

(3) $\left\{x \in \mathbb{R} : 1 \leq x^2 - 3x + 3 \leq \frac{7}{4}\right\}$

(4) $\left\{x \in \mathbb{R} : \frac{-x^2 + 4}{2x^2 - x - 1} \geq 0\right\}$

(5) $\left\{x \in \mathbb{R} : \frac{-x^2 - x + 2}{x^2 - 9} \leq 0\right\}$

(6) $\{x \in \mathbb{R} : \sqrt{3(x^2 - 1)} < 5 - x\}$

(7) $\{x \in \mathbb{R} : \sqrt{2x + 4} > x - 2\}$

(8) $\{x \in \mathbb{R} : \sqrt{x^2 - 1} > x + 3\}$

(9) $\left\{x \in \mathbb{R} : \sqrt{5x - 2} - \sqrt{x} < -\frac{1 - 3x}{\sqrt{x}}\right\}$

(10) $\{x^2 - 5x + 6 : x \in [0, 4]\}$

(11) $\{|x - 1| + 2|x| : x \in [-4, 2]\}$

(12) $\{x^2 - 5x + 6 : x^2 - 5x + 4 < 0\}$

(13) $\{x^2 - 5x + 4 : \sqrt{2x + 4} > x - 2\}$

(14) $\left\{\frac{1}{x} : \frac{x^2 + x - 2}{x^2 - 9} \leq 0\right\}$

(15) $\left\{x \in \mathbb{R} : 3^x > \frac{1}{27}\right\}$

(16) $\left\{4^x : 2^x < 40\right\}$

(17) $\left\{x^2 - 1 : \left(\frac{1}{4}\right)^{x^2+2} \leq 15\right\}$

(18) $\left\{x \in \mathbb{R} : 2^{|x-1|} < 2^x\right\}$

(19) $\left\{x > 0 : 3^{|x-1|} > 1\right\}$

(20) $\left\{x \in [-1, 3) : \left(\frac{1}{2}\right)^{x-2} > \left(\frac{1}{2}\right)^{x^2}\right\}$

(21) $\left\{x < 4 : \frac{2^x - 1}{2^x - 3} > 2^x\right\}$

(22) $\left\{x+4 : \left(\frac{1}{2}\right)^{3+x^2} \geq \left(\frac{1}{2}\right)^{4x}\right\}$

(23) $\left\{x^2-3 : \log_5(4|x|-x^2) < 1\right\}$

(24) $\left\{x \in \mathbb{R} : 0 < \log_{10}\left(\frac{x+2}{x+1}\right) < 1\right\}$

(25) $\left\{2^x : \log_{11}(x+5) + \log_{11}(x-2) < \log_{11}(3x-1)\right\}$

(26) $\left\{x \in \mathbb{R} : 2\log_{0,7}(x+1) - \log_{0,7}(x-1) > \log_{0,7}(3x-1)\right\}$

(27) $\left\{x > -4 : \log_{10}\left(\frac{2x+1}{x+3}\right) > 1\right\}$

(28) $\left\{x \in \mathbb{R} : \log_{10}(2x+1) - \log_{10}(x+3) \leq 1\right\}$

(29) $\left\{x \in \mathbb{R} : \log_{(x+1)}(x^2-4x+5) > 1\right\}$

(30) $\left\{x \in [0, 6\pi] : \sin x = \frac{\sqrt{2}}{2}\right\}$

(31) $\left\{x \in [-\pi, 6\pi] : \cos x = \frac{1}{2}\right\}$

(32) $\left\{x \in [0, 7\pi] : \operatorname{tg} x = \sqrt{3}\right\}$

(33) $\left\{x \in [-\pi, 4\pi] : \sin^2 x + 3\cos^2 x + \sin x - 2 = 0\right\}$

(34) $\left\{x \in [-\pi, 9\pi] : \cos(2x) + 2\sin^2 x + \cos^3 x - 2\cos x - 1 = 0\right\}$

(35) $\left\{x \in [-2\pi, 7\pi] : 1 + \cos^2 x - \sin x = 0\right\}$

(36) $\left\{x \in [-2\pi, 5\pi] : \cos(2x)(\cos^2 x - \sin x - 2) = 0\right\}$

(37) $\left\{x \in [-3\pi, 4\pi] : \cos x + \sin x = \sqrt{2}\right\}$

(38) $\left\{x \in [-4\pi, +\infty) : \cos x + \sqrt{3}\sin x = 0\right\}$

(39) $\left\{x \in [-\pi, \pi] : |\cos x - 1| < \cos x\right\}$

(40) $\left\{x \in [-\pi, \pi] : \cos x + \sqrt{3}\sin x + 1 \geq 0\right\}$

(41) $\left\{x \in [0, 2\pi] : \cos x + \sin(2x) > 0\right\}$

(42) $\left\{x \in [-\pi, \pi] : \frac{2 - \sin x - \sin^2 x}{\sin x \cos x} > 0\right\}$

(43) $\left\{x \in [-2\pi, 6\pi] : \sin^2 x + 3\cos^2 x + \sin x - 2 \geq 0\right\}$

(44) $\left\{x \in [-\pi, 5\pi] : 2\cos^2 x - \sin(2x) = \sqrt{2} + 1\right\}$

(45) $\left\{x \in [-3\pi, 4\pi] : \sqrt{3}\cos x + 3\sin x \geq 0\right\}$

(46) $\left\{x \in [-4\pi, 4\pi] : (2 - \sqrt{3})\cos x + \sin x \geq 1\right\}$

Esercizio 2. Determinare il dominio di definizione delle seguenti funzioni

(1) $f(x) = \sqrt{x^2 + 5x + 4}$

(2) $f(x) = \sqrt{x^2 - 2x + 1}$

(3) $f(x) = \sqrt{2 - x^2}$

(4) $f(x) = \sqrt{\dfrac{-x^2 + 4}{2x^2 - x - 1}}$

(5) $f(x) = \sqrt{x - 2|x| + 2}$

(6) $f(x) = \sqrt{\sqrt{2x + 4} - x + 2}$

(7) $f(x) = \sqrt{40 - 2^x}$

(8) $f(x) = \sqrt{\sqrt{3}\cos x + 3\sin x}$

(9) $f(x) = \sqrt{\cos x + \sqrt{3}\sin x + 1}$

(10) $f(x) = \log_3(4x - 1)$

(11) $f(x) = \log_3(x^2 - 2)$

(12) $f(x) = \sqrt{\log_3(x^2 - 2)}$

(13) $f(x) = \log_3(\sqrt{x^2 - 2})$

(14) $f(x) = \log_5(|x + 3| - 2)$

(15) $f(x) = \log_5(x^2 - 2|x| - 3)$

(16) $f(x) = \log_3\left(\sqrt{x^2 - 1} - x - 3\right)$

(17) $f(x) = \log_5\left(5 - x - \sqrt{3(x^2 - 1)}\right)$

(18) $f(x) = \log_3\left(3^x - \dfrac{1}{27}\right)$

(19) $f(x) = \log_5\left(2^x - 2^{|x-1|}\right)$

(20) $f(x) = \log_3\left(\cos x - |\cos x - 1|\right)$

(21) $f(x) = \mathrm{tg}(3x)$

(22) $f(x) = \arcsin\Big(\dfrac{x}{x+2}\Big)$

(23) $f(x) = \arccos\Big(\dfrac{x+1}{x^2-1}\Big)$

(24) $f(x) = \operatorname{arctg}\Big(\dfrac{3x}{x^2-4}\Big)$

(25) $f(x) = \arcsin(x^2 - 5x + 5)$

(26) $f(x) = \arcsin\big(x^2 + |x+1|\big)$

(27) $f(x) = \arccos\big(|2x^2 - 16x + 31|\big)$

(28) $f(x) = \arcsin\big(3^{|x-1|}\big)$

(29) $f(x) = \arcsin\big(\log_5(4|x| - x^2)\big)$

(30) $f(x) = \arcsin\Big(\log_{10}\Big(\dfrac{x+2}{x+1}\Big)\Big)$

Esercizio 3. Calcolare i seguenti numeri complessi

(1) $\dfrac{(1+2i)^2 - (1-i)^3}{(3+2i)^3 - (2+i)^2}$

(2) $(-1 + i\sqrt{3})^{60}$

(3) $(2 - 2i)^7$

(4) $(\sqrt{3} - 3i)^6$

(5) $\Big(\dfrac{1 + i\sqrt{3}}{1 - i}\Big)^{40}$

(6) $\Big(\dfrac{1-i}{1+i}\Big)^8$

(7) $\dfrac{(1+i)^9}{(1-i)^7}$

(8) $\dfrac{(1+i)(3-3i)}{(\sqrt{2} + i\sqrt{6})^5}$

(9) $\sqrt{i}$

(10) $\sqrt{2 - 2i\sqrt{3}}$

(11) $\sqrt[3]{i}$

(12) $\sqrt[3]{-1+i}$

(13) $\sqrt[4]{-1}$

(14) $\sqrt[4]{-i}$

(15) $\sqrt[4]{1}$

(16) $\sqrt[4]{1-i}$

(17) $\sqrt[5]{\sqrt{3}+i}$

Esercizio 4. Determinare tutte le soluzioni delle seguenti equazioni

(1) $z^2+4z+5=0$

(2) $z^2+2iz+3=0$

(3) $z^2-2z+1-i=0$

(4) $z^2-2z-i\sqrt{3}=0$

(5) $z^2+2(2\sqrt{3}+i)z+7=0$

(6) $z^3+1=0$

(7) $z^3+z^2+z+1=0$

(8) $z^4-4z^2+8=0$

(9) $z^4-2iz^2-2=0$

(10) $\left(\dfrac{2z+1}{2z-1}\right)^4=1$

(11) $z^5-1=0$

(12) $(z+1)^5-(z-1)^5=0$

(13) $z^6+27=0$

(14) $z^8=\dfrac{1+i}{\sqrt{3}-i}$

Esercizio 5. Utilizzare il principio di induzione per dimostrare le seguenti affermazioni

(1) $\sum_{k=1}^n k \equiv 1+2+\ldots+n=\frac{1}{2}\,n(n+1), \quad \forall n\in\mathbb{N}$

(2) $\sum_{k=1}^n k^2 \equiv 1+2^2+\ldots+n^2=\frac{1}{6}\,n(n+1)(2n+1), \quad \forall n\in\mathbb{N}$

(3) $\sum_{k=1}^n k^3 \equiv 1+2^3+\ldots+n^3=\frac{1}{4}\,n^2(n+1)^2, \quad \forall n\in\mathbb{N}$

(4) $\sum_{k=0}^n q^n \equiv 1+q+q^2+\ldots+q^n=\frac{q^{n+1}-1}{q-1}, \quad \forall q\neq 1, n\in\mathbb{N}$

(5) $\sum_{k=1}^n (8k-5)=n(4n-1), \quad \forall n\in\mathbb{N}$

(6) $\sum_{k=1}^n (2n+2k-1)=3n^2, \quad \forall n\in\mathbb{N}$

(7) $n!\geq 2^{n-1}$, $n\in\mathbb{N}$, $n\geq 2$

(8) $n!\geq 2\cdot 3^{n-2}$, $n\in\mathbb{N}$, $n\geq 2$