Course Proposal

Representations of affine and quantum affine algebras

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The course will cover topics in the representation theory of infinite dimensional Lie algebras and their quantum analogs. The first two or three weeks of the course will deal with the representation theory of simple Lie algebras. We will discuss well–established results and the methods and techniques which are used in the theory. In the rest of the course, we shall focus on integrable representations of affine Lie algebras. There are essentially two very different families of such representations: the ones where the center of the affine Lie algebra acts by a positive integer, called positive level representations, and those where it acts trivially called the level zero representations. The level zero representations include finite–dimensional representations of the affine algebra and in recent years there has been a lot of research activity in this direction. One of the goals of the course is to present this material in a systematic way and to end with an overview of the current trends in the subject. These include the connections with categorification and cluster algebras, through the work of Hernandez-Leclerc and Nakajima and to the ongoing recent work of Kashiwara and his co-authors on the relationship with quiver Hecke algebras. The course will also focus on the homological properties of the non–semisimple category of finite–dimensional representations of affine and quantum affine algebras and discuss my recent work on BGG reciprocity and the ideas of affine highest categories which are being developed by Kleshchev.

The course will be accessible to graduate students who have had a basic semester course in the structure theory of semisimple Lie algebras and some elementary representation theory. For instance, familiarity with the first few chapters of the book,

J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory.*

would be useful, although I will present a quick overview of the material. I will then discuss the construction of the Chevalley–Kostant integral form of the universal enveloping algebra of the simple Lie algebra and discuss some of the consequences and results for representations in characteristic p and the notion of highest weight categories of Cline, Parshall and Scott. A possible reference is:

E. Cline, B. Parshall and L. Scott, *Finite dimensional algebras and highest weight categories*, Crelle, (1988), 85–99.

The final topic on simple Lie algebras that we will discuss is the famous Bernstein– Gelfand–Gelfand category $\mathcal{O}$. We shall be interested mainly in the homological properties of $\mathcal{O}$ and the connections with highest weight categories. A reference for this part of the course could be,

J. E. Humphreys, *Representations of the semisimple Lie algebra in the BGG category $\mathcal{O}$*, Grad. Stud. Math. 94, American Mathematical Society.

Moving on, we shall recall the definition of affine Lie algebra and discuss the theory of highest weight integrable representations for affine Lie algebras. The analog of the integral form for the universal enveloping algebra of the affine Lie algebra was proved by H. Garland. We shall see that this allows to prove that the highest weight representations also have a lattice and hence can be defined over a field of characteristic p. Many of the interesting questions studied for simple Lie algebras can be asked in this context as well and should have interesting answers, although relatively little is known at this point. The relevant references here include,

V. Kac, *Infinite–dimensional Lie Algebras*, Cambridge University Press.

H. Garland, *The arithmetic theory of loop algebras,* J. Algebra, (1978), 480–551.

The final part of the course will be devoted to the theory of finite–dimensional representations of untwisted quantum affine algebras and the connections with positive level representations of affine Lie algebras. The material here will be taken from several papers, a selected few are given below. The course will end with a discussion of some open problems.

J. Beck, *Braid Group Action and Quantum Affine Algebras*, Comm.Math.Phys. 1994.

V. Chari and A. Pressley, *Quantum Affine Algebras*, Comm. Math. Physics, 1990.

V. Chari and D. Hernandez, *Beyond Kirillov Reshethkin modules*, Contemporary Mathematics,

V. Chari and B. Ion, *BGG reciprocity for current algebras* arXiv:1307.1440.

E. Frenkel and E. Mukhin, Combinatorics of q-characters of finite-dimensional representations of quantum affine algebras, Comm. Math. Phys. 2001.

E. Mukhin and C.A.S. Young, *Extended T-systems*, Selecta, 2012.

D. Hernandez, A. Moura, and F. Pereira, *Minimal affinizations of type D*, preprint.

D. Hernandez and B. Leclerc, *Quantum Grothendieck rings and derived Hall algebras*, to appear in Crelle, arXiv:1109.0862.

S. Kang, M. Kashiwara M. Kim and Se. Oh, Symmetric quiver Hecke algebras and R-matrices of quantum affine algebras, arXiv:1304.0323

H. Nakajima *Quiver varieties and cluster algebras*, Kyoto J. Math.2011.