$$F(x) = \int_0^x \underbrace{\left(\operatorname{arctg}(e^t - 1) - |t|\right)}_{f(t)} dt$$

$$\operatorname{dom}(F) = \mathbb{R}$$

$$\lim_{x \to +\infty} F(x) = \int_0^{+\infty} f(t)\, dt = -\infty$$

INTEGRALE IMPROPRIO DIVERGENTE

$$\lim_{x \to -\infty} F(x) = \int_0^{-\infty} f(t)\, dt =$$

$$= -\int_{-\infty}^{0} f(t)\, dt = +\infty$$

$$\lim_{x \to +\infty} \frac{F(x)}{x} = \lim_{x \to +\infty} \frac{\int_0^x f(t)\,dt}{x} =$$

$$\overset{H}{=} \lim_{x \to +\infty} f(x) = -\infty$$

NO ASINTOTO OBLIQUO $x \to +\infty$

Ex. $x \to -\infty$

$$F'(x) = f(x) = \operatorname{arcTg}(e^x - 1) - |x|$$

$$f(x) = \operatorname{arcTg}(e^x - 1) - |x|$$

$$\operatorname{dom}(f) = \mathbb{R}$$

$$\lim_{x \to \pm\infty} f(x) = -\infty$$

$$f(x) \underset{x\to+\infty}{=} -x + \operatorname{arcTg}\left(e^{x}(1+o(1))\right)$$

$$= -x + \pi/2 + o(1),$$

ASINTOTO OBLIQUO $x \to +\infty$ $\quad y = -x + \pi/2$

$$f(x) \underset{x\to-\infty}{=} x + \operatorname{arcTg}(e^{x}-1) =$$

$$x + \operatorname{arcTg}(-1+o(1)) =$$

$$= x - \pi/4 + o(1).$$

ASINTOTO OBLIQUO $x \to -\infty$ $\quad y = x - \pi/4$

$$f(x) = \operatorname{arcTg}(e^{x}-1) - |x|$$

$$f'(x) = \frac{e^{x}}{1+(e^{x}-1)^{2}} - \operatorname{sgn}(x), \quad x \neq 0$$

$$\operatorname{sgn}(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$

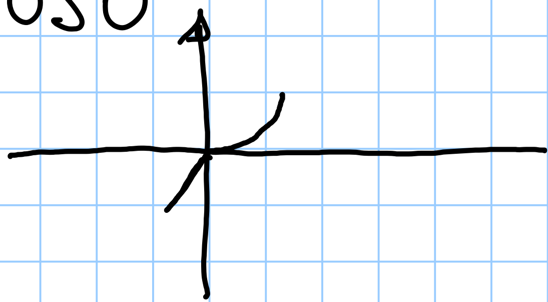
$\operatorname{sgn}(0) = 0$

$$D^{(+)}f(0) = \lim_{x\to 0^+} f'(x) = 1 - 1 = 0$$

$$D^{(-)}f(0) = \lim_{x\to 0^-} f'(x) = 1 + 1 = 2$$

$x = 0$ PUNTO ANGOLOSO

$$f(0) = 0$$



$$f'(x) > 0$$

$$\begin{cases} \dfrac{e^x}{1+(e^x-1)^2} - 1 > 0 & x > 0 \\ \dfrac{e^x}{1+(e^x-1)^2} + 1 > 0 & x < 0 \end{cases}$$

$\boxed{\text{SEMPRE}}$

$$\frac{e^x}{1+(e^x-1)^2} > 1 \quad , \quad x > 0$$

$$e^x > 1 + (e^x - 1)^2$$

$$e^x > 1 + e^{2x} - 2e^x + 1$$

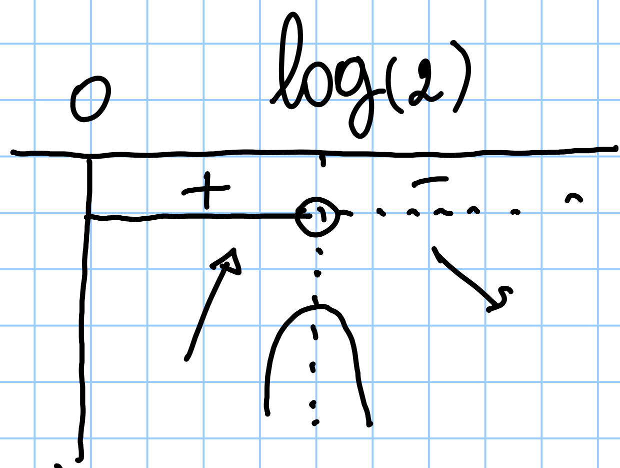
$$e^{2x} - 3e^x + 2 < 0$$

$$y = e^x$$

$$y^2 - 3y + 2 < 0$$

$$y_\pm = \frac{3 \pm \sqrt{9-8}}{2} = \frac{3 \pm 1}{2}$$

$$y_\pm \begin{cases} \nearrow 2 \\ \searrow 1 \end{cases}$$

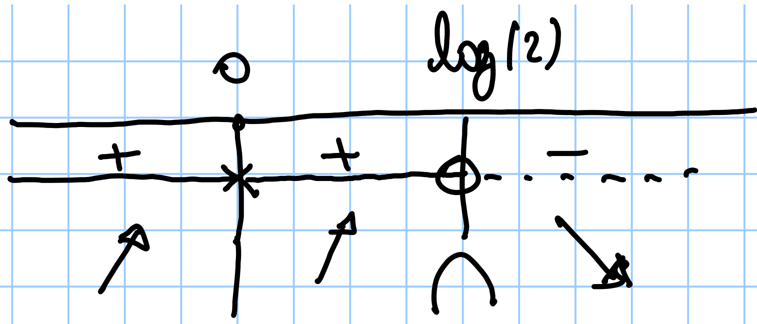



$$1 < y < 2$$

$$1 < e^x < 2$$

$$0 < x < \log(2)$$

$$f(\log(2)) = \frac{\pi}{4} - \log(2);$$




$x = \log(2)$

MASSIMO RELATIVO

$x \neq 0$

$$f''(x) = \frac{e^x(2-2e^x+e^{2x}) - (-2e^x+2e^{2x})e^x}{(1+(e^x-1)^2)^2}$$

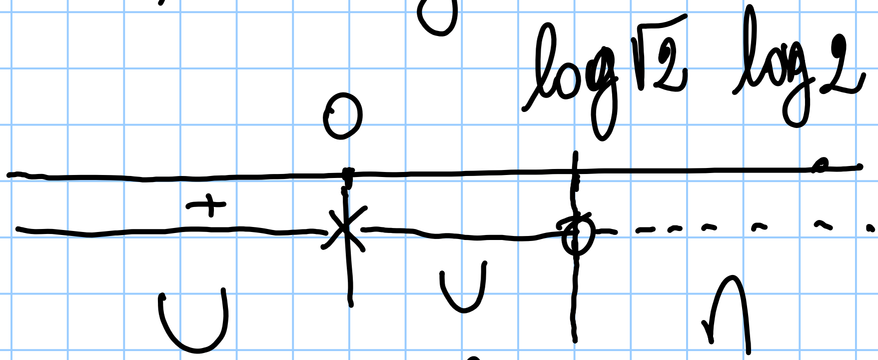
$$= \frac{e^x}{(\quad '' \quad)^2}\left[2-2e^x+e^{2x}+2e^x-2e^{2x}\right]$$

$$= \frac{e^x}{(\quad '' \quad)^2}\left[2-e^{2x}\right], \quad x \neq 0$$

$$2-e^{2x}>0$$

$$e^{2x}<2, \quad x<\frac{1}{2}\log(2)$$

$x < \log\sqrt{2}$




$x = \log(\sqrt{2})$

$x = \log(\sqrt{2})$

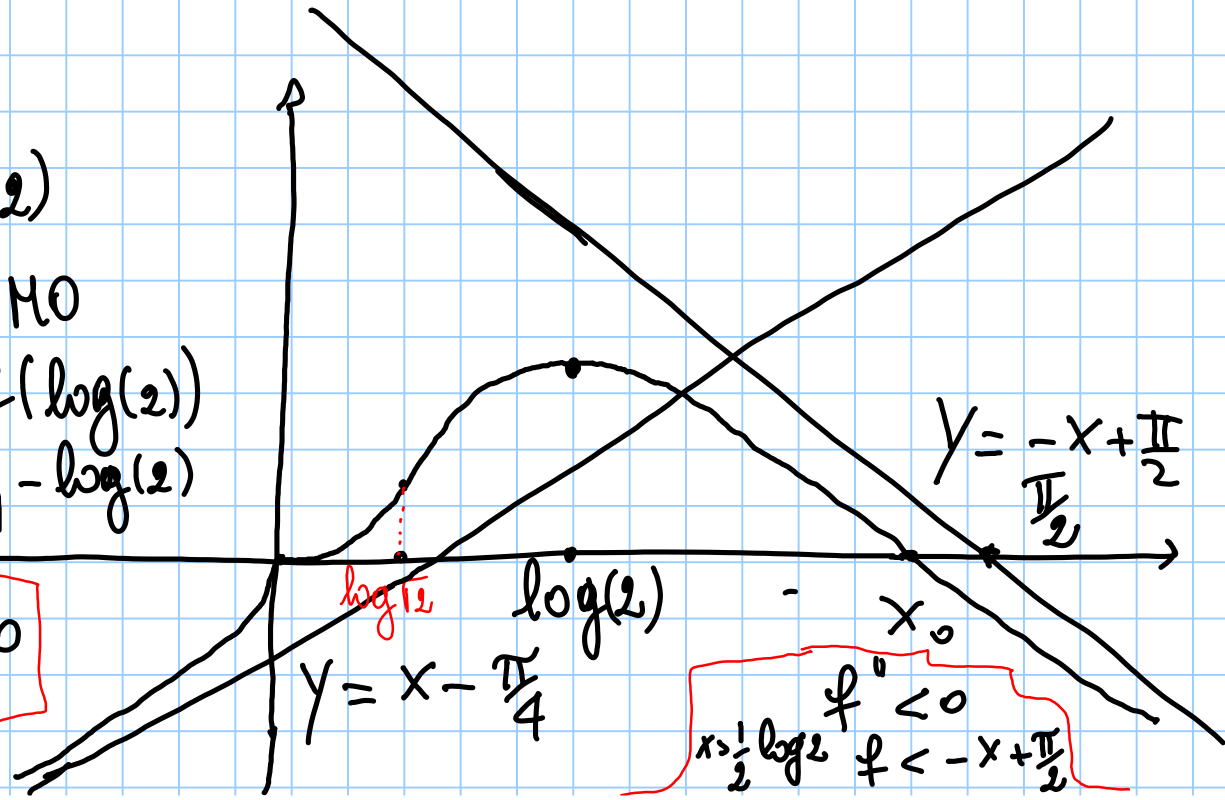
FLESSO ASCENDENTE

$f'(\log(\sqrt{2})) > 0$

$x = \log(2)$

MASSIMO

$\max_{\mathbb{R}} f = f(\log(2)) = \frac{\pi}{4} - \log(2)$




$x < 0 \quad f'' > 0$

$f > x - \frac{\pi}{4}$

$x > \frac{1}{2}\log 2 \quad f' < 0$

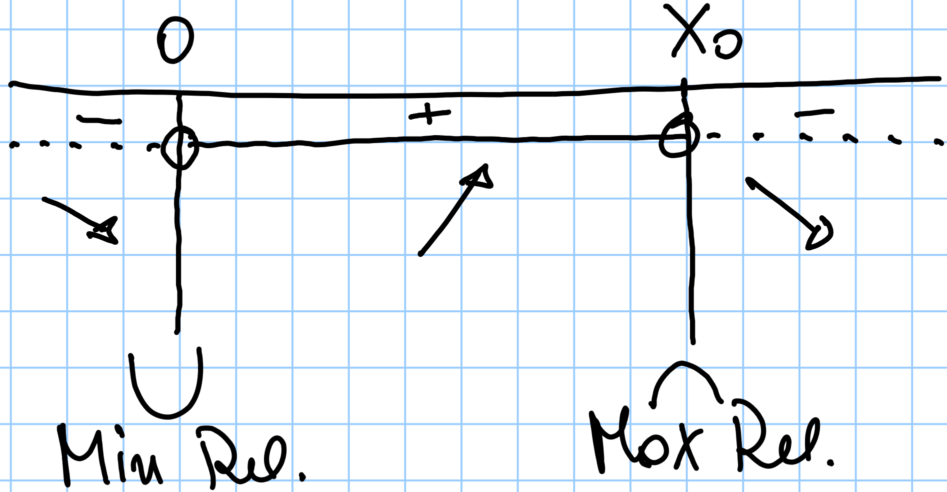
$f < -x + \frac{\pi}{2}$

$\text{arcTg}(e^x - 1) = |x|$ HA 2 SOLUZIONI

$x = 0 \quad , \quad \log 2 < x_0 < \frac{\pi}{2}$

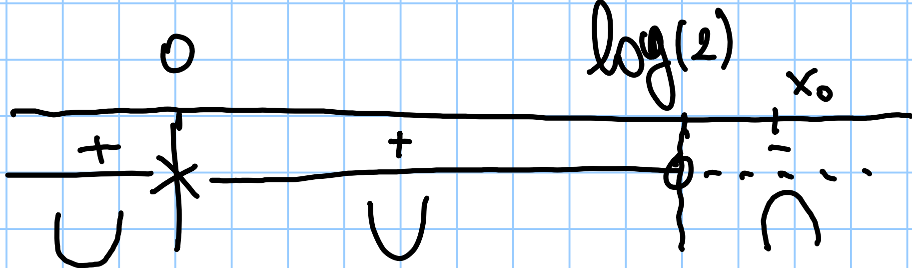
$F'(x) = \text{arcTg}(e^x - 1) - |x|$

$x = 0 \to F'(0) = 0$

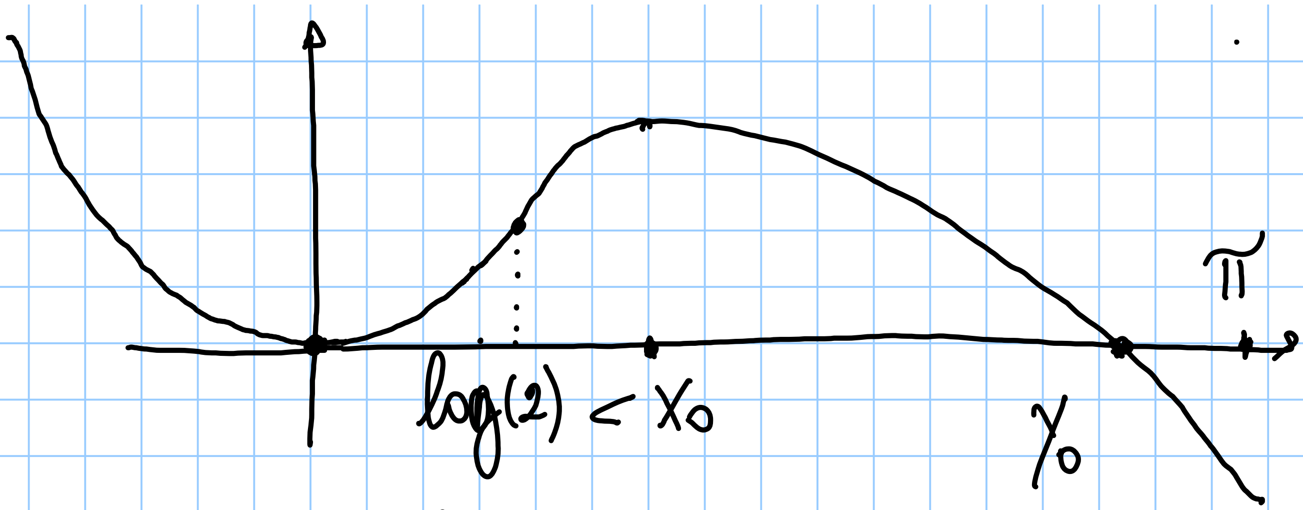



$F''(x) = f'(x)$

$F'(\log(2)) =$
$f(\log(2)) > 0$




$x = \log(2)$ FLESSO ASCENDENTE




$$F'(\log(2)) > 0$$

$$\log(2) < x_0 < y_0 < \pi$$

$$F(\pi) = \int_0^{\pi} \left( \operatorname{arctg}(e^t - 1) - |t| \right) dt$$

$$< \int_0^{\pi} \left( \frac{\pi}{2} - t \right) dt = \pi \cdot \frac{\pi}{2} - \frac{\pi^2}{2} = 0$$

$$F(x) = \int_0^{g(x)} f(t)\, dt$$

$$G(y) = \int_0^{y} f(t)\, dt$$

$$F(x) = G(g(x))$$

$$F'(x) = \dot{G}(g(x)) \cdot g'(x)$$

$$\boxed{\dot{G}(y) = \frac{d}{dy} G(y)} \qquad = f(g(x)) \cdot g'(x)$$

$$F(x) = \int_{h(x)}^{g(x)} f(t)\, dt$$

$$F'(x) = f(g(x))\, g'(x) - f(h(x))\, h'(x)$$

$$F(x)=\int_{-x^2}^{x^2}\frac{t e^{t}}{\sqrt{1-e^{-t^2}}}\,dt$$

$$f(t)=\frac{t e^{t}}{\sqrt{1-e^{-t^2}}}$$

$$|f(t)| = \frac{|t|\,e^{t}}{\sqrt{\cancel{1}-\left(\cancel{1}-t^2+o(t^2)\right)}} = \frac{|t|\,e^{t}}{|t|\,(1+o(1))} =$$

$$\left\{ \sqrt{t^2+o(t^2)} = \sqrt{t^2\,(1+o(1))} = |t|\sqrt{1+o(1)} = |t|(1+o(1)) \right\}$$

$$= 1+o(1)\ ,\quad t \to 0 .$$

L'INT. IMPROPRIO E'
CONVERGENTE $\forall\, x \in \mathbb{R}$
PERCHE' $|f|$ SI ESTENDE
PER CONTINUITA' IN $x=0$

$$|\tilde{f}(x)| = \begin{cases} |f(x)|\ , & x \neq 0 \\ 1\ , & x = 0 \end{cases}$$

$$F(x) = \int_{-x^2}^{x^2} f(t)\,dt$$

$$\mathrm{dom}(F) = \mathbb{R},$$

$$F(-x) = \int_{-(-x)^2}^{(-x)^2} f(t)\,dt = F(x)$$

F è PARI

$$F(0) = 0$$

$$\lim_{x \to \pm\infty} F(x) = +\infty$$

Ex

$$\lim_{x\to+\infty} \frac{F(x)}{x} \overset{H}{=} \lim_{x\to+\infty} \frac{F'(x)}{1} =$$

$$\lim_{x\to+\infty} \frac{x^2 e^{x^2}}{\sqrt{1-e^{-x^4}}} \cdot (2x) + \frac{x^2 e^{-x^2}(-2x)}{\sqrt{1-e^{-x^4}}}$$

$$= \lim_{x\to+\infty} \frac{2x^3}{\sqrt{1-e^{-x^4}}} \left(e^{x^2} - e^{-x^2}\right)$$

$= +\infty$ NO ASINTOTO OBLIQUO

Ex $\quad \dfrac{F(x)}{x^2 e^{x^2}} \longrightarrow 1, \quad x \to +\infty$

$$F'(x) = \frac{4x^3 \sinh(x^2)}{\sqrt{1-e^{-x^4}}}$$

$$\lim_{x\to 0} F'(x) = \lim_{x\to 0} \frac{4x^3 \cdot x^2 (1+o(1))}{\sqrt{1-(1-x^4+o(x^4))}} =$$

$$= \lim_{x\to 0} 4\frac{x^5}{x^2}(1+o(1)) = 0$$

$$F'(x) > 0 \quad \forall x > 0$$

$$F''(x) = 4\left[\frac{3x^2 \sinh(x^2)}{\sqrt{1-e^{-x^4}}} + \frac{x^3\cosh(x^2)\cdot 2x}{\sqrt{1-e^{-x^4}}} + \right.$$

$$\left. x^3\sinh(x^2)\left(-\frac{1}{2}\frac{(-e^{-x^4}\cdot)\cdot(-4x^3)}{(1-e^{-x^4})^{3/2}}\right)\right].$$

$$= 4\left[\frac{3x^2\sinh(x^2)}{(1-e^{-x^4})^{1/2}} + 2x^4\frac{\cosh(x^2)}{(1-e^{-x^4})^{1/2}} - 2x^6\frac{\sinh(x^2)\,e^{-x^4}}{(1-e^{-x^4})^{3/2}}\right] =$$

$$= \frac{4x^2}{(1-e^{-x^4})^{3/2}}\Big[3(1-e^{-x^4})\sinh(x^2) + 2x^2(1-e^{-x^4})\cosh(x^2) - 2x^4 e^{-x^4}\sinh(x^2)\Big] =$$

$$= \frac{4x^2}{(1-e^{-x^4})^{3/2}}\Big[\big(3-3e^{-x^4}-2x^4e^{-x^4}\big)\sinh(x^2) + \underbrace{2x^2(1-e^{-x^4})}_{\geq 0}\cosh(x^2)\Big]$$

$$3-3e^{-x^4}-2x^4e^{-x^4}\geq 0 \ ?$$

$$x^4=t,\quad t\geq 0$$

$$3-3e^{-t}-2te^{-t}\geq 0 \ ?$$

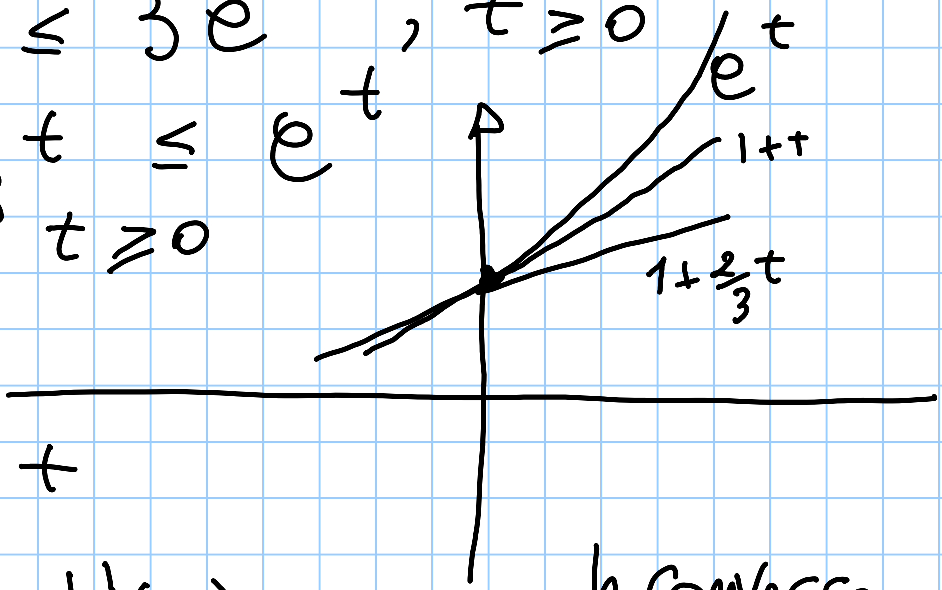
$$3-e^{-t}(3+2t)\geq 0$$

$$(3+2t)e^{-t}\leq 3,\quad t\geq 0$$

$$3+2t\leq 3e^{t},\quad t\geq 0$$

$$1+\frac{2}{3}t\leq e^{t}$$

$$t\geq 0$$




$$h(t)=e^{t}$$

$$y=h(0)+h'(0)t$$

$$1+t$$

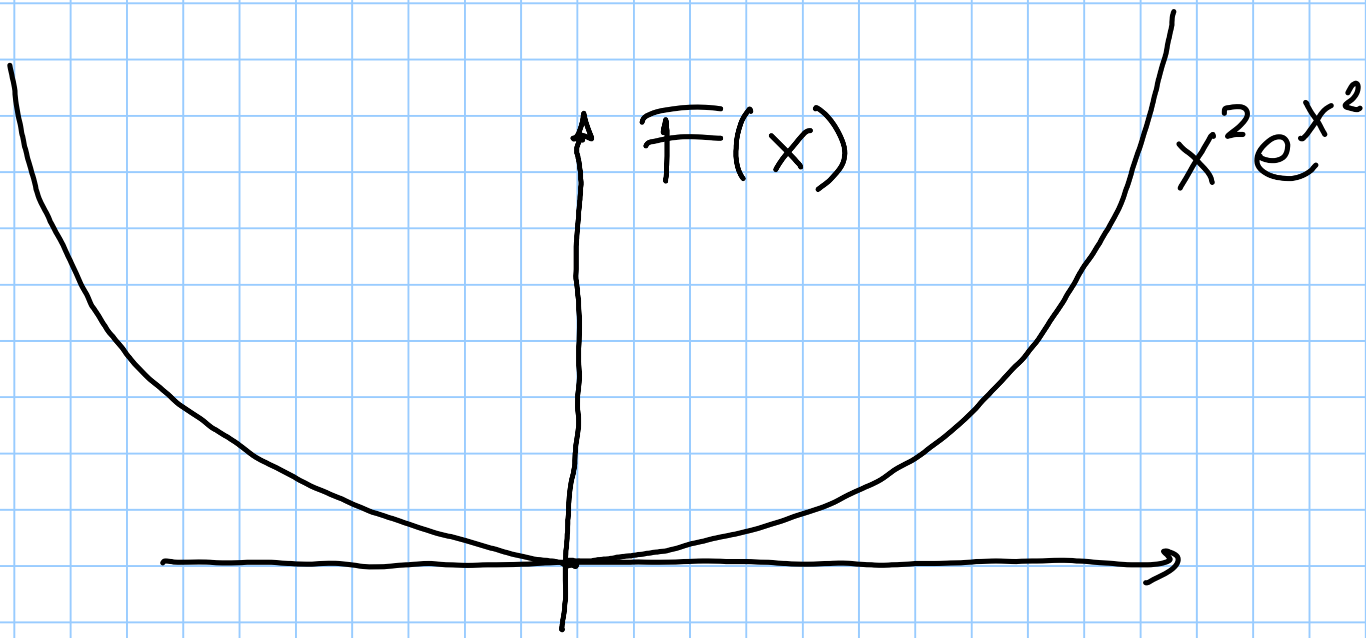
RETTA TANGENTE in $t=0$

h convessa

$$e^{t}\geq 1+t>1+\frac{2}{3}t$$

VERO $\forall\, t>0$.

$F''(x) > 0 \quad \forall\, x > 0$




Ex $\quad \dfrac{F(x)}{x^4} \xrightarrow[x \to 0]{} c \quad , \; c \neq 0$