EQUAZIONI DIFFERENZIALI

$$y(x) = 3e^{2x} \qquad x \in \mathbb{R}$$

$$y'(x) = 6e^{2x}$$

$$y'(x) = 2\,y(x) \qquad x \in \mathbb{R}$$

$\lambda \in \mathbb{R}\setminus\{0\}$, VOGLIAMO TROVARE

$y(x)$, DERIVABILE in $\mathbb{R}$:

EQUAZIONE DIFFERENZIALE

$$y'(x) = \lambda\, y(x), \; x \in \mathbb{R} \quad (1)$$

METODO di SEPARAZIONE DELLE VARIABILI

$$\frac{dy}{dx} = \lambda y \longrightarrow \text{"} \frac{dy}{y} = \lambda\, dx \text{"}$$

$$\int \frac{dy}{y} = \lambda \int dx$$

$$\log|y| = \lambda x + c_0$$

$$|y| = e^{c_0} e^{\lambda x}$$

$$y(x) = \pm e^{c_0} e^{\lambda x}$$

$$\boxed{y(x) = c\, e^{\lambda x} \quad c \in \mathbb{R}}$$

FORMULA GENERALE PER LA SOLUZIONE di (1)

NOTA: CASO $c = 0 \longrightarrow$

$y(x) \equiv 0 \quad \forall x \in \mathbb{R}$

LA FUNZIONE IDENTICAMENTE

NULLA È SOLUZIONE di (1)

ESISTONO INFINITE SOLUZIONI di (1)

$$y'(x) = \lambda y(x) \quad , \quad x \in \mathbb{R}$$

$$y(x) = 4e^{2x}$$

$$y'(x) = 8e^{2x} = 2y(x)$$

$$y(x) = -5e^{2x}$$

$$y'(x) = -10e^{2x} = 2y(x)$$

$$y(x) \equiv 0 \quad \forall x$$

$$y'(x) = 0 = 2y(x)$$

FISSANDO UN $\boxed{\text{DATO INIZIALE}}$

SI TROVA UNA SOLA SOLUZIONE

$$\begin{cases} y'(x) = 3y(x), & x \in \mathbb{R} \\ y(0) = 4 \end{cases}$$

$$y(x) = ce^{3x}, \quad x \in \mathbb{R}$$

$$y(0) = 4$$

$$ce^{3\cdot 0} = 4 \Rightarrow c = 4$$

$$y(x) = 4e^{3x}$$

L'EQUAZIONE $y'(x) = \lambda y(x)$

E' UN ESEMPIO di

EQUAZIONE DIFFERENZIALE

PROBLEMA di CAUCHY
associato all'equazione
$y'(x) = \lambda y(x)$

$$(P) \begin{cases} y'(x) = \lambda y(x), & x \in \mathbb{R} \\ y(x_0) = y_0 \end{cases}$$

TROVARE $y : \mathbb{R} \to \mathbb{R}$, DERIVABILE
TALE CHE $y'(x) = \lambda y(x)$, $x \in \mathbb{R}$
e $y(x_0) = y_0$

SOLUZIONE: $y(x) = ce^{\lambda x}$

$$c e^{\lambda x_0} = y_0 \longrightarrow c = y_0 e^{-\lambda x_0}$$

$$\boxed{y(x) = y_0 e^{\lambda(x-x_0)}}$$

EQUAZIONI DIFFERENZIALI
del 1° ORDINE A
VARIABILI SEPARABILI

$$y'(x) = g(x) f(y(x))$$

$$g : D_1 \to \mathbb{R} \; ; \; f : D_2 \longrightarrow \mathbb{R}$$

(2) $\quad y' = \dfrac{1}{y} \quad , \quad f(y) = \dfrac{1}{y}$

$$f : \mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R}$$

UNA SOLUZIONE di (2)
E' (PER DEFINIZIONE)

UNA FUNZIONE y DEFINITA
IN UN INTERVALLO $I \subseteq \mathbb{R}$,
DERIVABILE e CHE VERIFICA

- $y(x) \in \mathbb{R} \setminus \{0\}$ $\forall x \in I$
- $y'(x) = \dfrac{1}{y(x)}$, $\forall x \in I$

"$y\,dy = dx$", $\int y\,dy = \int dx$

$$\frac{y^2}{2} = x + c \quad , \quad y^2 = 2x + 2c$$

$$y(x) = \pm \sqrt{2(x+c)}$$

IL SEGNO + o − e c e I

SI OTTENGONO FISSANDO UN DATO INIZIALE

$$\begin{cases} y' = \frac{1}{y} \\ y(2) = -1 \end{cases}$$

$$\pm\sqrt{2(2+c)} = -1$$

$y(2) = -1$

$$-\sqrt{2(2+c)} = -1$$

$$4 + 2c = 1 \longrightarrow c = -\frac{3}{2}$$

$$y(x) = -\sqrt{2x-3}$$

$$I = \left(\frac{3}{2}, +\infty\right)$$

$$y(x) < 0 \quad \forall x \in \left(\frac{3}{2}, +\infty\right)$$

$$\begin{cases} y' = \dfrac{\sqrt{2-x}}{2y} \\ y(1) = 3 \end{cases} \qquad (P)$$

UNA SOLUZIONE di (P) E' (PER DEFINIZIONE)

UNA FUNZIONE y DEFINITA IN UN INTERVALLO $I \subseteq (-\infty, 2]$ DERIVABILE e CHE VERIFICA

- $y(x) \in \mathbb{R} \setminus \{0\} \quad \forall x \in I$ $\left(\begin{matrix} y(x) \in D_2 \\ \forall x \in I \end{matrix} \right)$
- $2 - x \geq 0 \qquad (I \subseteq D_1)$
- $y'(x) = \dfrac{\sqrt{2-x}}{y(x)} \qquad \forall x \in I$
- $1 \in I$ e $y(1) = 3$

$$2\int y\,dy = \int \sqrt{2-x}\,dx$$

$$y^2 = -\frac{2}{3}(2-x)^{3/2} + c$$

$$-\frac{2}{3}(2-1)^{3/2} + c = (y(1))^2 = 3^2$$

$$c = 9 + \frac{2}{3} = \frac{29}{3}$$

$$y^2 = \frac{29}{3} - \frac{2}{3}(2-x)^{3/2}$$

$$y(x) = \pm\sqrt{\frac{29}{3} - \frac{2}{3}(2-x)^{3/2}}$$

$y(1)=3$

$$y(x) = \sqrt{\frac{29}{3} - \frac{2}{3}(2-x)^{3/2}}$$

I ? $\quad (\; y(x) \neq 0 \;\; \forall x \in I \;)$

$$29 - 2(2-x)^{3/2} > 0$$

$$(2-x)^{3/2} < 29/2$$

$$\begin{cases} x \leq 2 \\ 2-x < \left(\frac{29}{2}\right)^{2/3} \end{cases}$$

$$\begin{cases} x \leq 2 \\ x > 2 - \left(\frac{29}{2}\right)^{2/3} \end{cases}$$

$$I = \left(2 - \left(\frac{29}{2}\right)^{2/3}, 2 \right]$$

INTERVALLO MASSIMALE

di ESISTENZA DELLA SOLUZIONE

OVVERO, dato

$$\begin{cases} y' = g(x)\,f(y) \\ y(x_0) = y_0 \end{cases} \qquad (P)$$

$g : D_1 \longrightarrow \mathbb{R}$, $f : D_2 \longrightarrow \mathbb{R}$

I = il più grande intervallo

TALE CHE:

- $y : I \to \mathbb{R}$, $x_0 \in I$;
- $I \subseteq D_1$;
- $y(x) \in D_2 \ \forall x \in I$;

• y VERIFICA (P)

$$\begin{cases} y' = y^2 \, Tg(x) \\ y(\pi) = 2 \end{cases}$$

$$\int \frac{dy}{y^2} = \int Tg(x)\, dx$$

$$-y^{-1}(x) = -\log|\cos(x)| + c$$

$$y^{-1}(x) = \log|\cos(x)| - c$$

$$\log|\cos(\pi)| - c = \frac{1}{y(\pi)} = \frac{1}{2}$$

$$C = -\frac{1}{2}$$

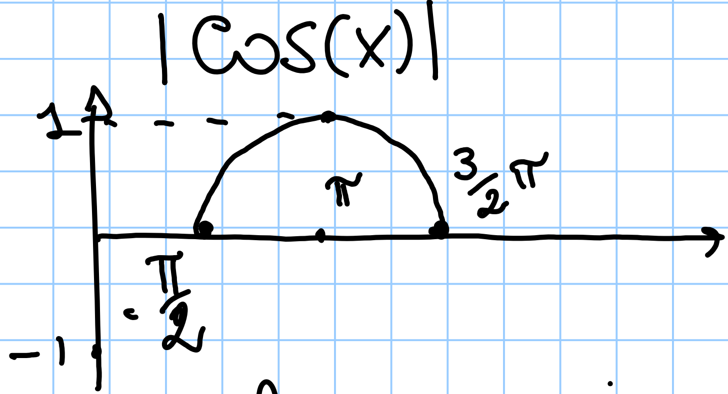
$$\frac{1}{y(x)} = \log|\cos(x)| + \frac{1}{2}$$

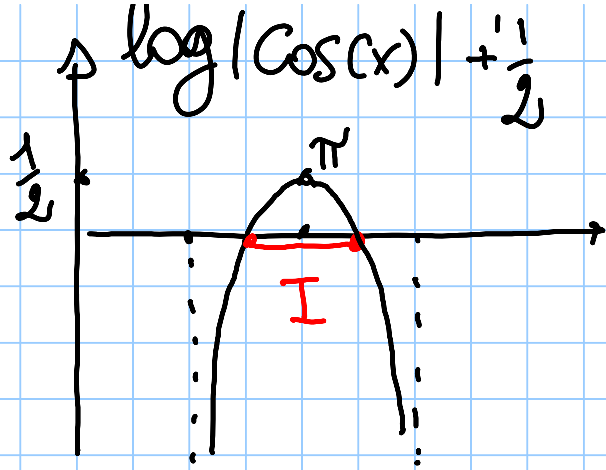
$$y(x) = \frac{1}{\log|\cos(x)| + \frac{1}{2}}$$

I ? $\quad \pi \in I$

$$\mathrm{dom}(\mathrm{Tg}(x)) = \bigcup_{k \in \mathbb{Z}} \left((2k-1)\cdot\frac{\pi}{2}, (2k+1)\frac{\pi}{2}\right)$$

$$I \subseteq \left(\frac{\pi}{2}, \frac{3}{2}\pi\right)$$





$$g(x) = \log|\cos(x)| + \frac{1}{2},$$

$$g(\pi) = \frac{1}{2}$$

$$I = \left\{ x \in \left(\frac{\pi}{2}, \frac{3}{2}\pi\right) : g(x) > 0 \right\}$$

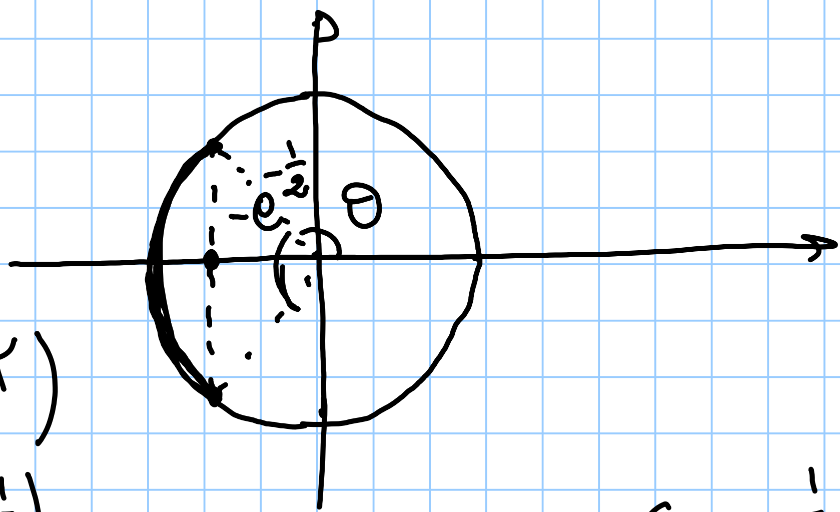
$$\log|\cos(x)| > -\frac{1}{2}$$

$$|\cos(x)| > e^{-\frac{1}{2}}$$

$$x \in \left(\frac{\pi}{2}, \frac{3}{2}\pi\right) \longrightarrow |\cos(x)| = -\cos(x)$$

$$-\cos(x) > e^{-\frac{1}{2}}$$

$$\cos(x) < -e^{-\frac{1}{2}}$$



$$\theta \in \left(\frac{\pi}{2}, \pi\right)$$

$$\arccos\left(-e^{-\frac{1}{2}}\right) < x < -\arccos\left(-e^{-\frac{1}{2}}\right) + 2\pi$$

$$2\pi - \theta \in \left(\pi, \frac{3}{2}\pi\right)$$

$$I = (\theta, 2\pi - \theta)$$

NOTA: I È più piccolo di $\left(\frac{\pi}{2}, \frac{3}{2}\pi\right)$

$$\begin{cases} y'(x) = \frac{5}{2}\sqrt[3]{(x+1)^2(y(x)+1)} \\ y(0) = 0 \end{cases}$$

$$y' = \frac{5}{2}(x+1)^{2/3}(y+1)^{\frac{1}{3}}$$

$$\int \frac{dy}{(y+1)^{1/3}} = \frac{5}{2}\int (x+1)^{\frac{2}{3}}\,dx$$

$$\frac{3}{2}(y+1)^{2/3} = \frac{5}{2}\,\frac{(x+1)^{5/3}}{5/3} + c$$

$$(y(x)+1)^{2/3} = (x+1)^{5/3} + c$$

$$(0+1)^{5/3} + c = (y(0)+1)^{2/3} = (0+1)^{2/3}$$

$$c = 0$$

$$(y(x)+1)^{2/3} = (x+1)^{5/3}$$

$$(y(x)+1)^{2} = (x+1)^{5}$$

$$y(x)+1 = \pm (x+1)^{5/2}$$

$y(0)=0$

$$y(x)+1 = (x+1)^{5/2}$$

$$y(x) = (x+1)^{5/2} - 1$$

$$I = [-1, +\infty)$$