$$\lim_{\substack{x \to 0^+ \\ x \to +\infty}} f(x) \;;$$

$$f(x) = \frac{(\operatorname{arcTg}(x))\left(2\sinh(x) + \log\left(x^2 + \cos(2\sqrt{x})\right)\right)}{x^2 + 2\left(1 + \sin(x) - e^x\right)}$$

$$f(x) \underset{x \to +\infty}{=} \left(\frac{\pi}{2} + o(1)\right) \frac{\left(e^x - e^{-x} + \log\left(x^2(1+o(1))\right)\right)}{x^2 - 2e^x(1 + o(1))}$$

$$= \frac{\pi}{2}(1+o(1)) \frac{e^x(1+o(1))}{-2e^x(1+o(1))} \xrightarrow[x \to +\infty]{} -\frac{\pi}{4}.$$

$$x^2 + 2\left(1 + \sin(x) - e^x\right) \underset{x \to 0}{=}$$

$$x^2+2\left(\cancel{1}+\cancel{x}-\frac{x^3}{3!}+o(x^4)\right)-\cancel{1}-\cancel{x}-\frac{x^2}{2}$$

$$-\frac{x^3}{3!}+o(x^3)\Big)\Big)=x^2-x^2-2\frac{x^3}{3!}-2\frac{x^3}{3!}$$

$$+o(x^3)=-\frac{4}{6}x^3+o(x^3)=-\frac{2}{3}x^3(1+o(1))$$

$$\left(2\sinh(x)+\log\left(x^2+\cos(2\sqrt{x})\right)\right)=$$

$$2(x+o(x^2))+\log\left(x^2+1-\frac{4}{2}x+\frac{2^4x^2}{4!}+\right.$$

$$\left.+o\left((\sqrt{x})^5\right)\right)=2x+o(x^2)+$$

$$\log\left(1-2x+\left(1+\frac{2^4}{4\cdot3\cdot2}\right)x^2+o(x^{5/2})\right)$$

$$=2x+o(x^2)+\log\left(1-2x+\frac{5}{3}x^2+o(x^{5/2})\right)$$

$$=2x+o(x^2)+\left(-2x+\frac{5}{3}x^2+o(x^{5/2})\right)$$

$$-\frac{1}{2}\left(-2x+\frac{5}{3}x^2+o(x^{5/2})\right)^2+o(x^2)=$$

$$= 2x - 2x + \frac{5}{3}x^2 - 2x^2 + o(x^2) =$$

$$= -\frac{1}{3}x^2\,(1 + o(1))$$

$$f(x) \underset{x\to 0}{=} \frac{x(1+o(1))\left(-\frac{1}{3}x^2\right)}{-\frac{2}{3}x^3} \xrightarrow[x\to 0]{} \frac{1}{2}\,.$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$f(x) = \operatorname{arcTg}\left(x + \log|x-2| \right)$$

$$\operatorname{dom}(f) = \mathbb{R} \setminus \{2\}$$

$$\lim_{x \to +\infty} f(x) = \left(\frac{\pi}{2}\right)^-;$$

$$\lim_{x \to -\infty} f(x) = \left(-\frac{\pi}{2}\right)^+;$$

ASINTOTI ORIZZONTALI

$y = \frac{\pi}{2}$, $x \to +\infty$

$y = -\frac{\pi}{2}$, $x \to -\infty$

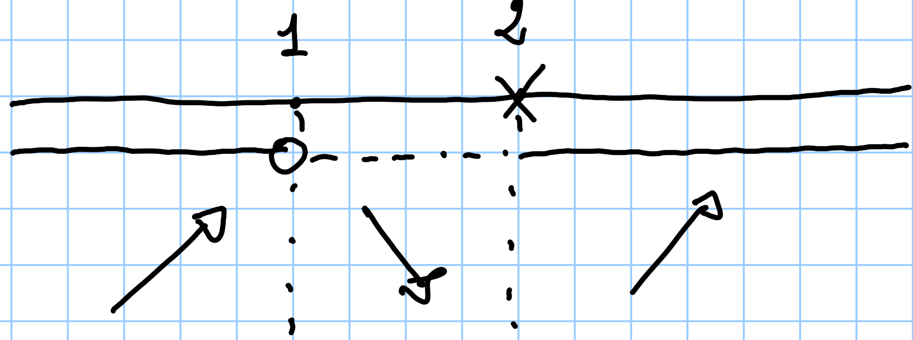
$$\lim_{x \to 2} f(x) = \left(-\frac{\pi}{2}\right)^+$$

f si estende per continuità in $x_0 = 2$, $f(2) = -\frac{\pi}{2}$

$$f'(x) = \frac{1 + \frac{\operatorname{sgn}(x-2)}{|x-2|}}{\left(1+(x+\log|x-2|)^2\right)} =$$

$$= \frac{1 + \frac{1}{x-2}}{\left(1+(x+\log|x-2|)^2\right)} , \quad x \neq 2$$

$$= \frac{1}{\left(1+(x+\log|x-2|)^2\right)} \cdot \frac{x-1}{x-2}$$



$x = 1$ massimo LOCALE

$$f(1) = \operatorname{arcTg}\left(1 + \log|-1|\right) = \frac{\pi}{4}$$

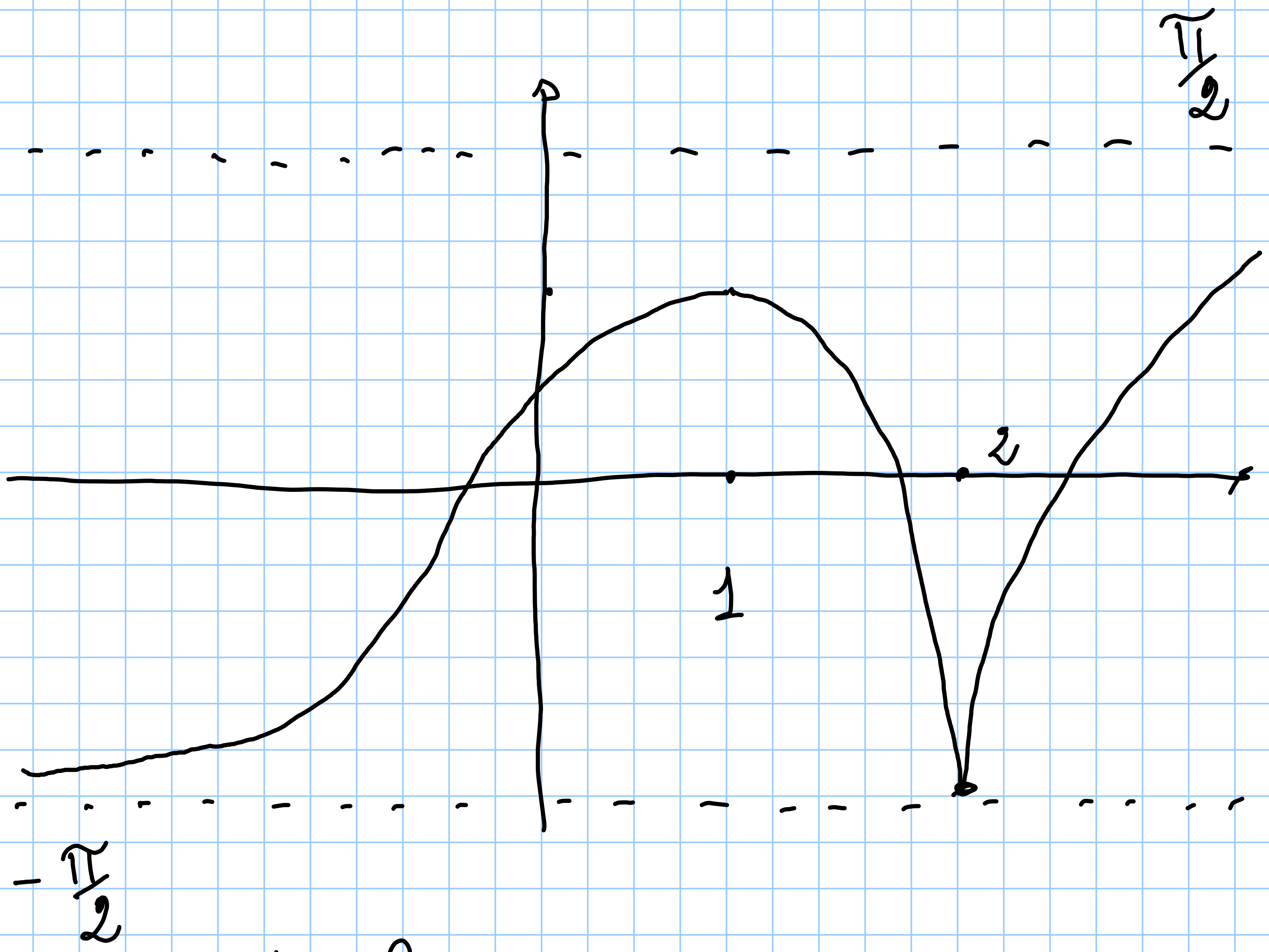
$x = 2$ minimo LOCALE

$$f(2) = -\frac{\pi}{2}$$

$$f'(x) \xrightarrow{x \to 2^{\pm}} = \frac{1 + o(1)}{\left(\log|x-2|\right)^2} \frac{1}{x-2} \to$$

$$\to \frac{1}{0^{\pm}} = \pm\infty$$

$x = 2$ PUNTO di CUSPIDE



$\min f = -\pi/2$

$\sup f = \pi/2$

T_4 $\quad f(x) = \log\left(1 + (\log(x))^2\right)$

$x_0 = 1$

$x = 1 + y$

$g(y) = \log\left(1 + \log^2(1+y)\right)$

T_4 , $y_0 = 0$.

$\log(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} + o(y^3)$

$\log^2(1+y) = y^2 - y^3 + \frac{2}{3}y^4 + o(y^4)$

$+ \frac{y^4}{4} = y^2 - y^3 + \frac{3+8}{12}y^4 + o(y^4)$

$= y^2 - y^3 + \frac{11}{12}y^4 + o(y^4)$

$$\log\left(1 + y^2 - y^3 + \frac{11}{12}y^4 + o(y^4)\right) =$$

$$= y^2 - y^3 + \frac{11}{12}y^4 + o(y^4) +$$

$$-\frac{1}{2}\left(y^2 - y^3 + o(y^3)\right)^2 + o\left(\left(y^2 - y^3 + o(y^3)\right)^2\right)$$

$$= y^2 - y^3 + \frac{11}{12}y^4 - \frac{1}{2}y^4 + o(y^4)$$

$$= y^2 - y^3 + \frac{5}{12}y^4 + o(y^4)$$

$$T_4(x) = (x-1)^2 - (x-1)^3 + \frac{5}{12}(x-1)^4$$

$$\lim_{n} \frac{\left(e^{-\frac{1}{2n^2}} - \cos\left(\frac{1}{n}\right)\right)^{n}\left((n+1)^{n} + \sqrt{n!}\right)^{4}}{12^{-n} + 4^{-2n}}$$

$$12^{-n} + 4^{-2n} = 12^{-n} + 16^{-n} =$$

$$= 12^{-n}\left(1 + \left(\frac{12}{16}\right)^{n}\right) =$$

$$= 12^{-n}\left(1 + o(1)\right)$$

$$(n+1)^{n} + \sqrt{n!} \quad = n^{n}\left[\left(1+\frac{1}{n}\right)^{n} + \frac{\sqrt{n!}}{n^{n}}\right]$$

$$= n^{n}\left[\, e + o(1) + \frac{\left(\cancel{n^{n}} e^{-n}\sqrt{2\pi(n)}\left(1+o(1)\right)\right)^{\frac{1}{2}}}{n^{n/2}} \right]$$

$$= n^{n}\left[\, e + o(1) \right]$$

$$e^{-\frac{1}{2n^2}} - \cos\left(\frac{1}{n}\right) = 1 - \frac{1}{2n^2} + \frac{1}{2}\left(\frac{1}{2n^2}\right)^2 +$$

$$+ o\left(\frac{1}{n^4}\right) - \left(1 - \frac{1}{2n^2} + \frac{1}{4!}\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)\right) =$$

$$= \left(\frac{1}{8} - \frac{1}{24}\right)\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)$$

$$\lim_n \frac{\left(\frac{1}{12}\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)\right)^n n^{4n}\left(e + o(1)\right)^4}{12^{-n}\left(1 + o(1)\right)} =$$

$1^{+\infty}$

$$= \lim_n \frac{\cancel{\frac{12^{-n}}{n^{4n}}}\left(1 + o(1)\right)^n \cancel{n^{4n}}\, e^4\left(1 + o(1)\right)}{\cancel{12^{-n}}\left(1 + o(1)\right)}$$

$$e^{-\frac{1}{2n^2}} - \cos\left(\frac{1}{n}\right) = 1 - \frac{1}{2n^2} + \frac{1}{2}\left(\frac{1}{2n^2}\right)^2 +$$

$$+ \frac{1}{3!}\left(-\frac{1}{2n^2}\right)^3 + o\left(\frac{1}{n^6}\right) - \left(1 - \frac{1}{2n^2} + \frac{1}{4!}\frac{1}{n^4} - \frac{1}{6!}\frac{1}{n^6}\right.$$

$$\left. + o\left(\frac{1}{n^6}\right)\right) =$$

$$\left(\frac{1}{8} - \frac{1}{24}\right)\frac{1}{n^4} - \frac{1}{3!8}\frac{1}{n^6} + \frac{1}{6!}\frac{1}{n^6} + o\left(\frac{1}{n^6}\right) =$$

$$= \frac{1}{12}\frac{1}{n^4} + \frac{a}{n^6} + o\left(\frac{1}{n^6}\right), \quad a = \frac{1}{6!} - \frac{1}{3!8}$$

$$\left(e^{-\frac{1}{2n^2}} - \cos\left(\frac{1}{n}\right)\right)^n =$$

$$\left(\frac{1}{12}\frac{1}{n^4} + \frac{a}{n^6} + o\left(\frac{1}{n^6}\right)\right)^n =$$

$$\frac{12^{-n}}{n^{4n}}\left(1+\frac{12a}{n^2}+o\left(\frac{1}{n^2}\right)\right)^{n} =$$

$$= \frac{12^{-n}}{n^{4n}}\left(1+\frac{12a+o(1)}{n^2}\right)^{\frac{n^2}{n}} =$$

$$= \frac{12^{-n}}{n^{4n}}\left[\left(1+\frac{12a+o(1)}{n^2}\right)^{n^2}\right]^{\frac{1}{n}}$$

$$\to \left(e^{12a}\right)^{0} = 1$$

$$= \lim_{n} \frac{\cancel{\frac{12^{-n}}{n^{4n}}}\left(1+o(1)\right)\, \cancel{n^{4n}}\, e^{4}\left(1+o(1)\right)}{\cancel{12^{-n}}\left(1+o(1)\right)} =$$

$$= e^{4}$$

$$f(x) = \operatorname{arcTg}\left(x\, e^{\frac{1}{x^2-1}}\right)$$

- $\operatorname{dom}(f) = \mathbb{R} \setminus \{-1, 1\}$
- f DISPARI

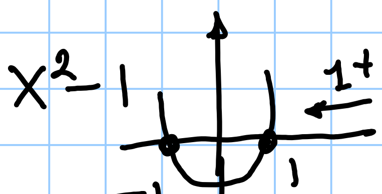
ASINTOTO ORIZZONTALE

$$f(0) = 0 \quad , \quad \lim_{x \to +\infty} f(x) = \frac{\pi}{2}$$

$$\left(\lim_{x \to -\infty} f(x) = -\frac{\pi}{2} \right)$$

$$\lim_{x \to 1^+} f(x) = \operatorname{arcTg}\left(1\, e^{\frac{1}{0^+}}\right) =$$

$$= \operatorname{arcTg}(+\infty) = \frac{\pi}{2}$$



$$\lim_{x \to 1^-} f(x) = \operatorname{arcTg}\left(1\, e^{\frac{1}{0^-}}\right) =$$

$$= \operatorname{arcTg}(0) = 0$$

$x = 1$ DISCONTINUITA' di SALTO

$$f'(x) = \frac{e^{\frac{1}{x^2-1}} + x e^{\frac{1}{x^2-1}} \left(\frac{-2x}{(x^2-1)^2} \right)}{1 + \left(x e^{\frac{1}{x^2-1}} \right)^2} =$$

$$= \frac{e^{\frac{1}{x^2-1}}}{\left(1 + \left(x e^{\frac{1}{x^2-1}}\right)^2\right)(x^2-1)^2} \cdot \left[(x^2-1)^2 - 2x^2 \right] =$$

$$= \quad // \quad \left[x^4 - 4x^2 + 1 \right]$$

$$x^2 = t; \qquad t^2 - 4t + 1 \geq 0$$

$$t_{\pm} = \frac{2 \pm \sqrt{4-1}}{1} = 2 \pm \sqrt{3}$$

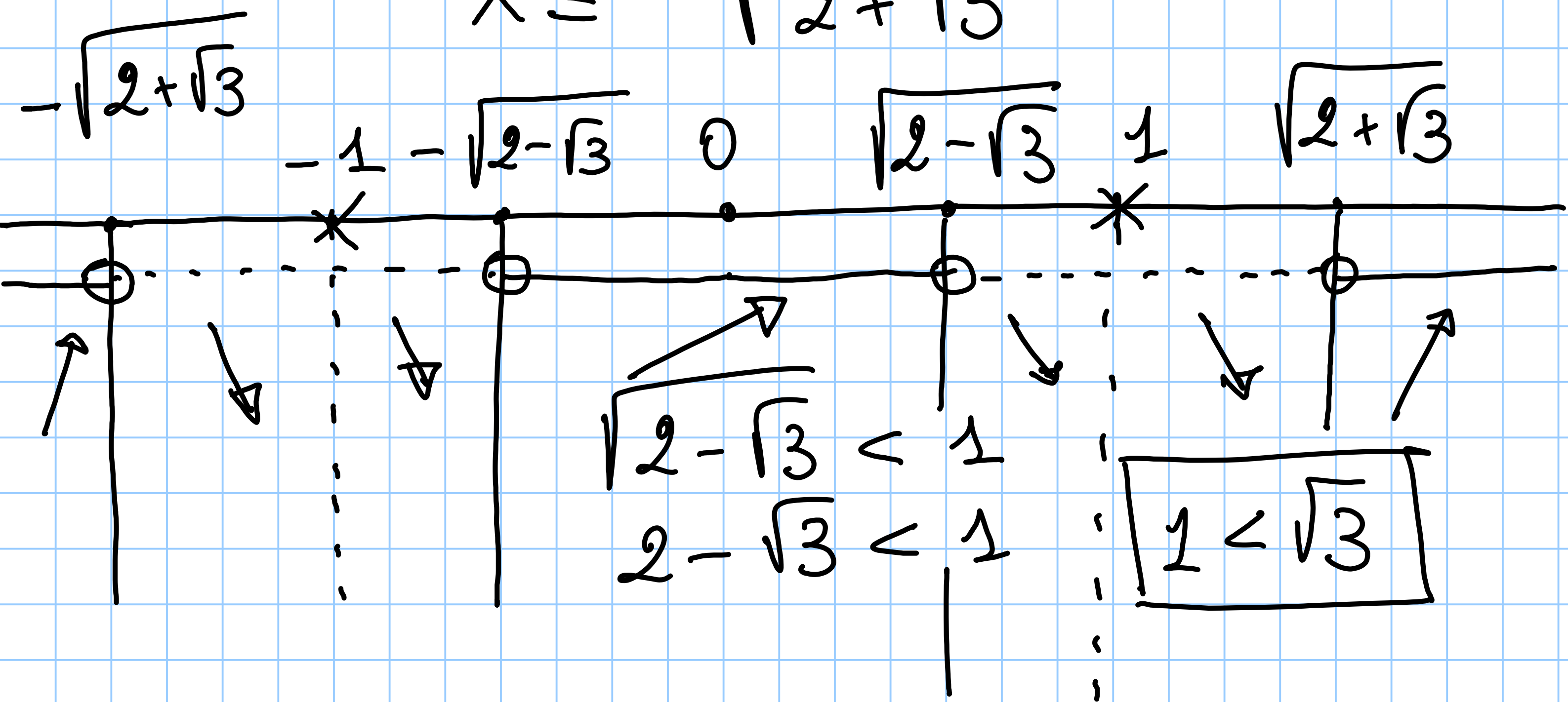
$t \leq 2-\sqrt{3} \quad , \quad t \geq 2+\sqrt{3}$

$x^2 \leq 2-\sqrt{3} \quad , \quad x^2 \geq 2+\sqrt{3}$

$-\sqrt{2-\sqrt{3}} \leq x \leq \sqrt{2-\sqrt{3}} \quad ,$

$x \geq \sqrt{2+\sqrt{3}}$

$x \leq -\sqrt{2+\sqrt{3}}$



$\sqrt{2-\sqrt{3}} < 1$

$2-\sqrt{3} < 1$

$\boxed{1 < \sqrt{3}}$

$x_2 = \sqrt{2+\sqrt{3}}$ MINIMO LOCALE

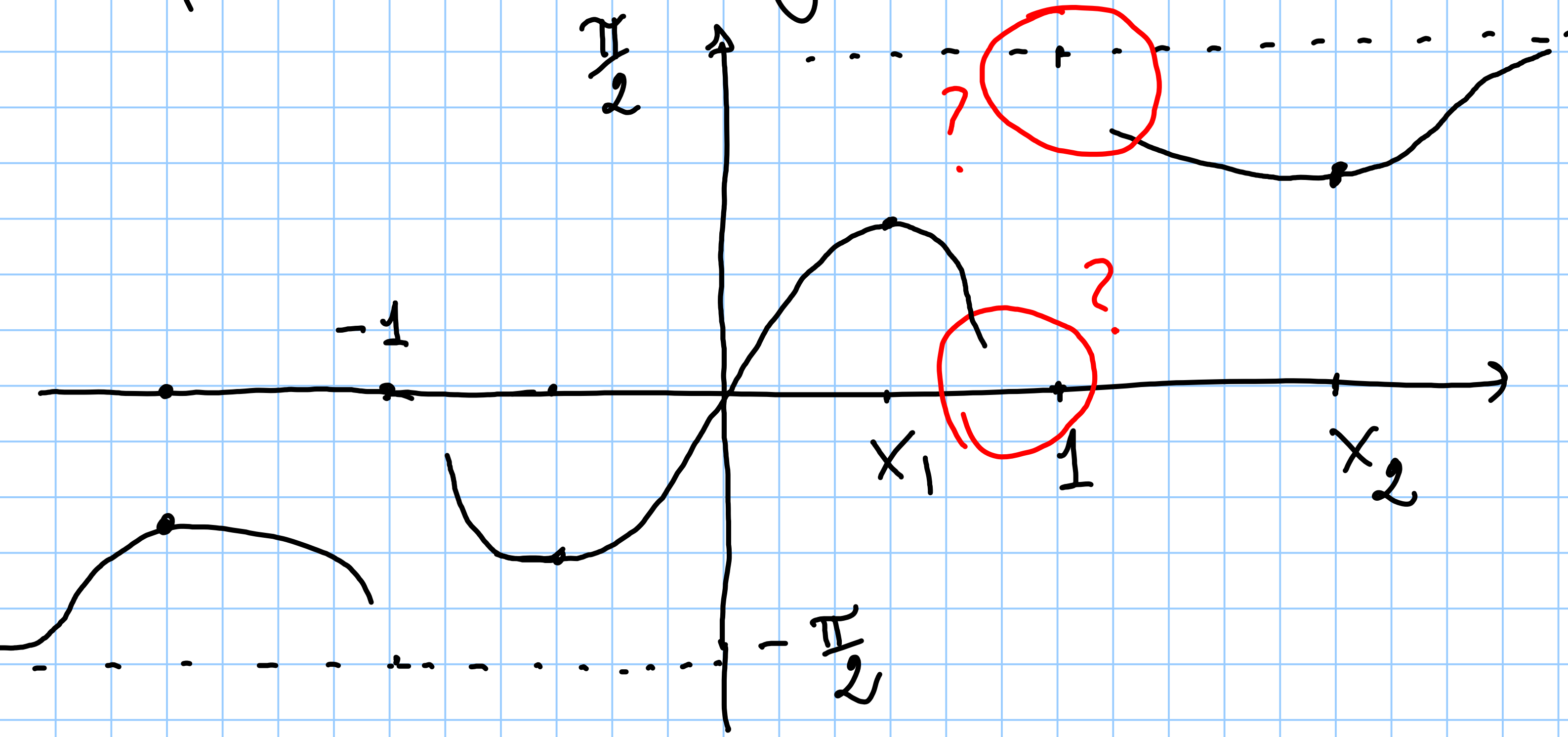
$x_1 = \sqrt{2-\sqrt{3}}$ MASSIMO LOCALE

$$\begin{pmatrix} -\sqrt{2+\sqrt{3}} & \text{MASSIMO LOCALE} \\ -\sqrt{2-\sqrt{3}} & \text{MINIMO LOCALE} \end{pmatrix}$$

$$x_1^2 = 2-\sqrt{3} \qquad x_1^2 - 1 = 1-\sqrt{3}$$

$$f(x_1) = \operatorname{arcTg}\left((2-\sqrt{3})^{\frac{1}{2}} e^{\frac{1}{1-\sqrt{3}}}\right)$$

$$f(x_2) = \operatorname{arcTg}\left((2+\sqrt{3})^{\frac{1}{2}} e^{\frac{1}{1+\sqrt{3}}}\right)$$



$$\overset{x \to 1^+}{} \quad f'(x) = \frac{e^{\frac{1}{x^2-1}}}{\left(1+\left(x e^{\frac{1}{x^2-1}}\right)^2\right)(x^2-1)^2}\left[-2x^2+o(1)\right]$$

$$= \frac{-2(1+o(1))}{(x^2-1)^2} \frac{e^{\frac{1}{x^2-1}}}{\left(x e^{\frac{1}{x^2-1}}\right)^2(1+o(1))} = -\frac{2(1+o(1))}{(x^2-1)^2 e^{\frac{1}{x^2-1}}}$$

Ex $\left(\underline{x^2-1=t} \right)$

$$(x^2-1)^2 e^{\frac{1}{x^2-1}} \xrightarrow{x \to 1^+} +\infty, \quad x \to 1^+$$

$$\lim_{x \to 1^+} f'(x) = \frac{-2}{+\infty} = 0^-$$

$x = 1$ PUNTO di TANGENZA ORIZZONTALE DA DESTRA

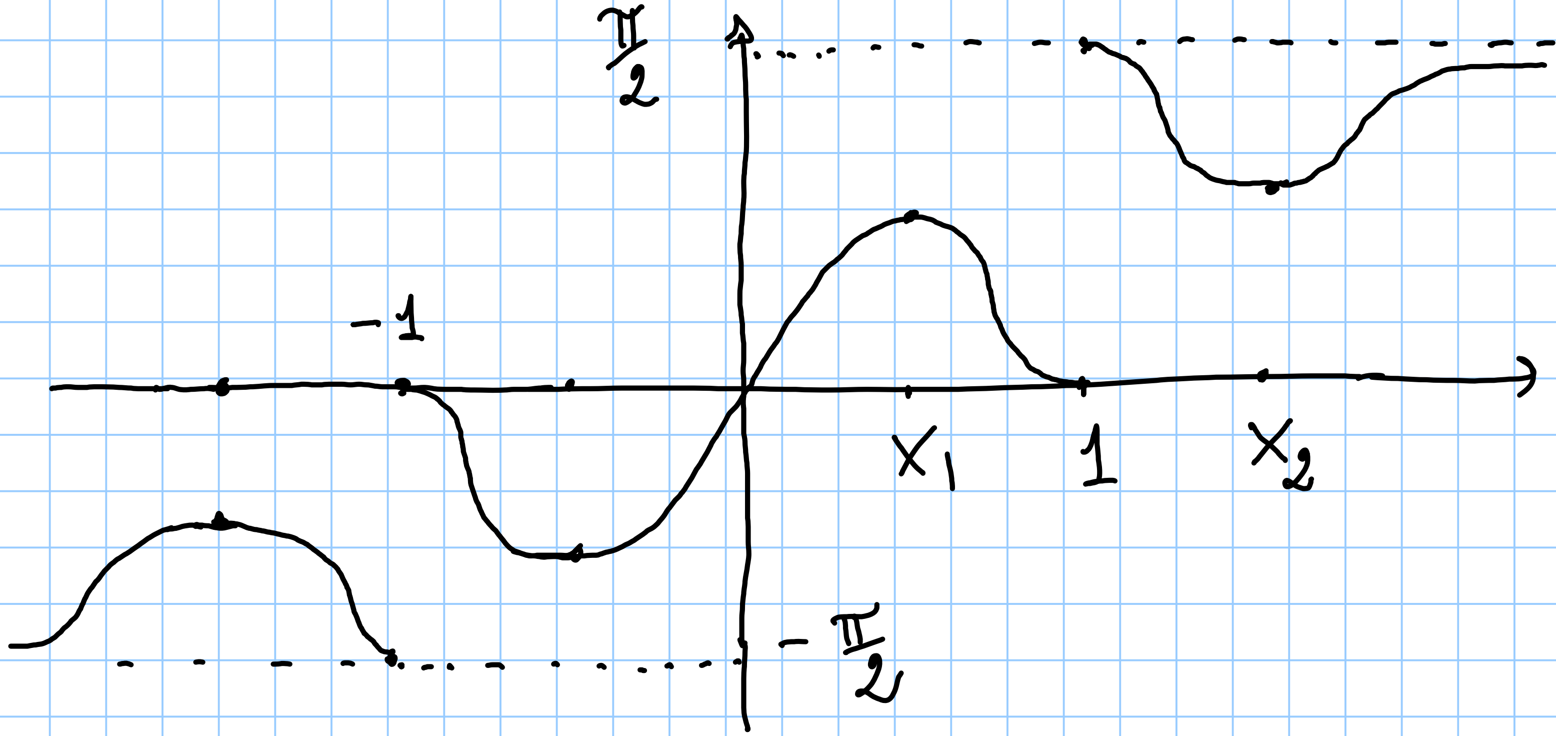
$$f'(x) \underset{x\to 1^-}{=} \frac{e^{\frac{1}{x^2-1}}}{\left(1+\left(xe^{\frac{1}{x^2-1}}\right)^2\right)(x^2-1)^2}\left[-2x^2+o(1)\right] =$$

$$= \frac{-2(1+o(1))}{(x^2-1)^2}\,\frac{e^{\frac{1}{x^2-1}}}{(1+o(1))} = -2\,\frac{e^{\frac{1}{x^2-1}}(1+o(1))}{(x^2-1)^2} \longrightarrow 0^-$$

$$\left\{ t = x^2-1 \quad , \quad t \to 0^- \quad , x \to 1^- \right\}$$

$$\lim_{x\to 1^-} \frac{e^{1/x^2-1}}{(x^2-1)^2} = \lim_{t\to 0^-} \frac{e^{1/t}}{t^2} = \left\{ y = -\frac{1}{t} \right\}$$

$$= \lim_{y\to +\infty} y^2 e^{-y} = 0^+$$


$\frac{\pi}{2}$
-1
x_1
1
x_2
$-\frac{\pi}{2}$

$$\lim_{n} \frac{\sqrt{n^4+n^2+\frac{1}{2}} - n^2 - \frac{1}{2}}{n^{\alpha}}$$

$$\lim_{n} \frac{n^2\sqrt{1+\frac{1}{n^2}+\frac{1}{2n^4}} - n^2 - \frac{1}{2}}{n^{\alpha}} =$$

$$\lim_{n} \frac{\sqrt{1+\frac{1}{n^2}+\frac{1}{2n^4}} - 1 - \frac{1}{2n^2}}{n^{\alpha-2}} =$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{2}\left(\frac{1}{2}-1\right)x^2 + o(x^2)$$

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + o(x^2)$$

$$\left(1 + \frac{1}{m^2} + \frac{1}{2m^4}\right)^{\frac{1}{2}} = 1 + \frac{1}{2m^2} + \frac{1}{4m^4} +$$

$$-\frac{1}{8}\left(\frac{1}{m^2} + o\left(\frac{1}{m^3}\right)\right)^2 + o\left(\frac{1}{m^4}\right) =$$

$$= 1 + \frac{1}{2m^2} + \frac{1}{4m^4} - \frac{1}{8m^4} + o\left(\frac{1}{m^4}\right)$$

$$= 1 + \frac{1}{2n^2} + \frac{1}{8n^4} + o\left(\frac{1}{n^4}\right)$$

$$\lim_{n} \frac{1 + \frac{1}{2n^2} + \frac{1}{8n^4} + o\left(\frac{1}{n^4}\right) - 1 - \frac{1}{2n^2}}{n^{\alpha-2}}$$

$$= \lim_{n} \frac{1}{8} \frac{1+o(1)}{n^{\alpha+2}} = \begin{cases} \alpha > -2 & 0^+ \\ \alpha = -2 & \frac{1}{8} \\ \alpha < -2 & +\infty \end{cases}$$

$$\lim_{x \to 0^+} \frac{\log(x + \cos(x)) - x + x^2}{x^\alpha}$$

$$\cos(x) = 1 - \frac{x^2}{2} + o(x^3)$$

$$\log\left(x + 1 - \frac{x^2}{2} + o(x^3)\right) =$$

$$= \log\Big(1 + \underbrace{x - \frac{x^2}{2} + o(x^3)}_{y}\Big) =$$

$$\log(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} + o(y^3)$$

$$= \left(x - \frac{x^2}{2} + o(x^3)\right) - \frac{1}{2}\left(x - \frac{x^2}{2} + o(x^3)\right)^2$$

$$+ \frac{1}{3}\left(x - \frac{x^2}{2} + o(x^3)\right)^3 + o(x^3) =$$

$$= x - \frac{x^2}{2} + o(x^3) - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{3}x^3$$

$$= x - x^2 + \frac{5}{6}x^3 + o(x^3)$$

$$\lim_{x \to 0^+} \frac{\log(x + \cos(x)) - x + x^2}{x^\alpha} =$$

$$\lim_{x \to 0^+} \frac{x - x^2 + \frac{5}{6}x^3 + o(x^3) - x + x^2}{x^\alpha} =$$

$$\lim_{x \to 0^+} \frac{5}{6}x^{3-\alpha}(1 + o(1)) = \begin{cases} 0^+ & \alpha < 3 \\ \frac{5}{6} & \alpha = 3 \\ +\infty & \alpha > 3 \end{cases}$$

$$\lim_{x \to 0^+} \frac{\log(x+\cos(x)) - x + x^2 - \frac{5}{6}x^3}{x^\alpha}$$

$$x + \cos(x) = x + 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)$$

$$\log(x+\cos(x)) =$$

$$\log\left(1 + x - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)\right) =$$

$$x - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4)$$

$$-\frac{1}{2}\left(x - \frac{x^2}{2} + o(x^3)\right)^2$$

$$+\frac{1}{3}\left(x - \frac{x^2}{2} + o(x^3)\right)^3 - \frac{1}{4}x^4 + o(x^4) =$$

$$= x - \frac{x^2}{2} + \frac{x^4}{24} - \frac{1}{2}x^2 + \frac{1}{2}x^3 - \frac{1}{2}\frac{x^4}{4}$$

$$+ \frac{1}{3}x^3 + \frac{1}{3}\left(-\frac{3}{2}x^4\right) - \frac{1}{4}x^4 + o(x^4)$$

$$= x - x^2 + \frac{5}{6}x^3 + x^4\left(\frac{1}{24} - \frac{1}{8} - \frac{1}{2} - \frac{1}{4}\right)$$

$$+ o(x^4) = x - x^2 + \frac{5}{6}x^3 + x^4\left(\frac{1-3-12-6}{24}\right)$$

$$+ o(x^4) = x - x^2 + \frac{5}{6}x^3 - \frac{5}{6}x^4 + o(x^4)$$

$$\lim_{x \to 0^+} \frac{-\frac{5}{6}x^4(1+o(1))}{x^\alpha} = \begin{cases} 0 & \alpha < 4 \\ -\frac{5}{6} & \alpha = 4 \\ -\infty & \alpha > 4 \end{cases}$$