$$\lim_{x \to 0^+} \frac{\left(1+x-\sin(x)\right)^{1/x} - 1}{e^{x+x^2} - 1 - x}$$

$$\left(1 + x - \sin(x)\right)^{1/x} \xrightarrow[x \to 0^+]{} 1^{+\infty} \ (\text{F.I.})$$

$$D(x) = e^{x+x^2} - 1 - x$$

$$e^{x+x^2} = 1 + x + x^2 + \frac{1}{2}(x+x^2)^2 + o(x^2) =$$

$$= 1 + x + x^2 + \frac{1}{2}x^2 + o(x^2)$$

$$= 1 + x + \frac{3}{2}x^2 + o(x^2)$$

$$D(x) = \frac{3}{2}x^2 + o(x^2)$$

$$\left(1+x-\sin(x)\right)^{1/x} =$$

$$= e^{\frac{1}{x}\log\left(1+x-\sin(x)\right)} =$$

$$= e^{\frac{1}{x}\log\left(1+x-x+\frac{x^3}{3!}+o(x^3)\right)} =$$

$$= e^{\frac{1}{x}\left(\frac{x^3}{3!}+o(x^3)\right)} = e^{\frac{x^2}{6}+o(x^2)} =$$

$$= 1+\frac{x^2}{6}+o(x^2)$$

$$\lim_{x\to 0^+}\frac{N(x)}{D(x)} = \lim_{x\to 0^+}\frac{\cancel{1}+\frac{x^2}{6}+o(x^2)-\cancel{1}}{\frac{3}{2}x^2+o(x^2)} = \frac{1}{9}$$

$$f(x) = \log\left(1 + e^{\frac{x^2}{|x|-1}}\right)$$

- $\mathrm{dom}(f) = \{x \in \mathbb{R} : |x| \neq 1\}$

$$= \mathbb{R} \setminus \{-1, +1\}$$

- f PARI $\Rightarrow x \in [0, +\infty) \setminus \{1\}$

- $1 + e^{\frac{x^2}{|x|-1}} > 1 \quad \forall x \notin \{-1, 1\}$

- LIMITI AGLI ESTREMI

$$f(0) = \log(1 + e^0) = \log(2)$$

$$f(x) \underset{x \to +\infty}{=} \log\left(1 + e^{\frac{x^2}{x-1}}\right) =$$

$$= \log\left(e^{x(1+o(1))} + 1\right) =$$

$$= \log e^{x(1+o(1))} + \log\left(1 + e^{-x(1+o(1))}\right)$$

$$= x + o(x), \quad x \to +\infty$$

ASINTOTO? OBLIQUO

$$\frac{x^2}{x-1} = \frac{x^2}{x} \frac{1}{1-\frac{1}{x}} = x \cdot \frac{1}{1-1/x} =$$

$$\underset{x \to +\infty}{=} x\left(1 + \frac{1}{x} + o\left(\frac{1}{x}\right)\right) = x + 1 + o(1)$$

$$\log\left(1 + e^{\frac{x^2}{x-1}}\right) \underset{x \to +\infty}{=} \log\left(1 + e^{x+1+o(1)}\right)$$

$$= \log\left(e^{x+1+o(1)}\right) + \log\left(1 + e^{-(x+1+o(1))}\right)$$

$$= x+1+o(1)+o(1), \quad x+1+o(1), \quad x \longrightarrow +\infty$$

ASINTOTO OBLIQUO

$$y = x+1, \quad x \to +\infty$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \log\left(1+e^{\frac{x^2}{x-1}}\right)$$

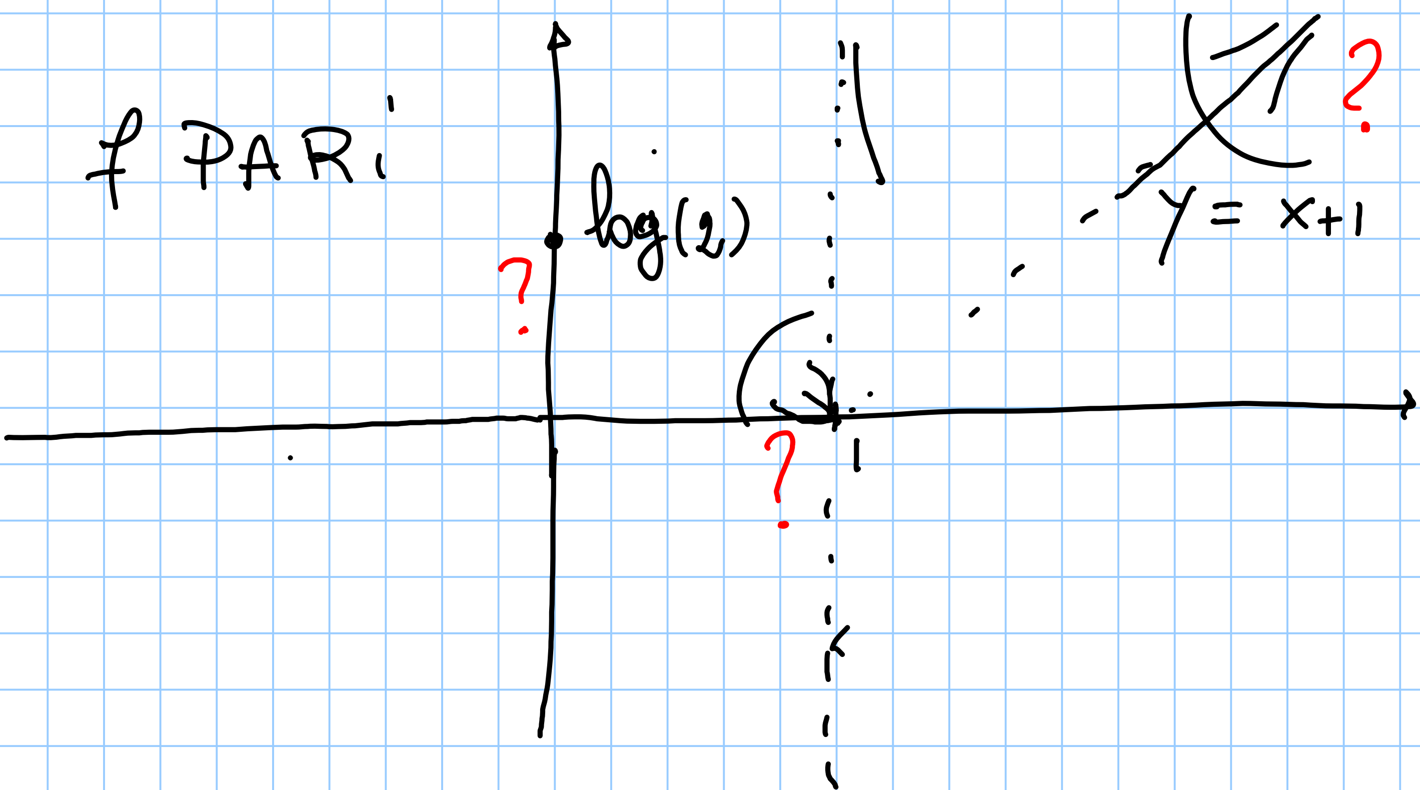
$$= \log\left(1+e^{\frac{1}{0^+}}\right) = +\infty$$

$$\lim_{x \to 1^-} f(x) = \log\left(1+e^{\frac{1}{0^-}}\right) =$$

$$= \log(1+0^+) = 0^+$$

$x = 1$ ASINTOTO VERTICALE (DESTRO)

f PARI

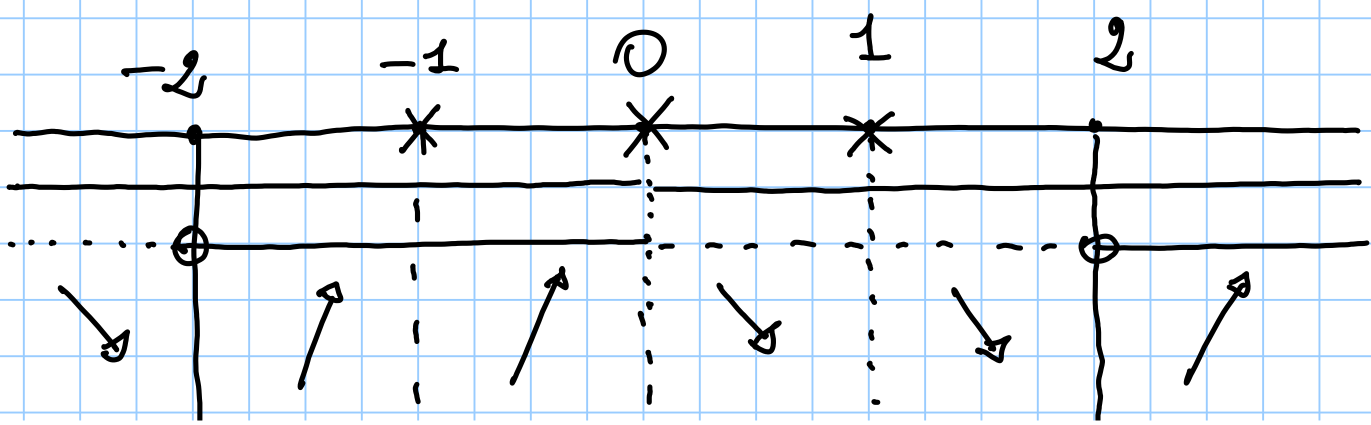


$$f'(x) = \frac{e^{\frac{x^2}{x-1}}}{\left(1+e^{\frac{x^2}{x-1}}\right)} \quad \frac{2x(x-1)-x^2}{(x-1)^2}, \quad \begin{matrix} x>0 \\ x\neq 1 \end{matrix}$$

$$\frac{e^{\frac{x^2}{x-1}}}{\left(1+e^{\frac{x^2}{x-1}}\right)} \quad \frac{x^2-2x}{(x-1)^2} \;;$$

$f'(x) > 0 \iff x^2-2x>0$;

$x>0, x\neq 1$ $\qquad \{x<0\} \cup \{x>2\}$



$f(x)$ PARI

$$f'(-x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h} =$$

$h = -t$

$$= \lim_{t \to 0} \frac{f(-x-t) - f(-x)}{-t} =$$

$$= -\lim_{t \to 0} \frac{f(x+t) - f(x)}{t} = -f'(x)$$

$x_0 = 0$ MASSIMO RELATIVO

$x = \pm 2$ MINIMI RELATIVI

$f(0) = \log(2)$; $f(\pm 2) = \log(1+e^4)$

$$\lim_{x\to 0^+} f'(x) = \frac{e^0}{1+e^0}\,\frac{0-0}{(-1)^2} = 0$$

f PARI

$$\lim_{x\to 0^-} f'(x) = -\lim_{x\to 0^+} f'(x) = 0$$

T. CONTINUITÀ DELLA f':

$$\lim_{x\to 0} f'(x) = 0 \Longrightarrow f'(0) = 0$$

f è DERIVABILE in $x_0 = 0$.

$$\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} \frac{e^{\frac{x^2}{x-1}}}{1+o(1)} \cdot \frac{x^2-2x}{(x-1)^2}$$

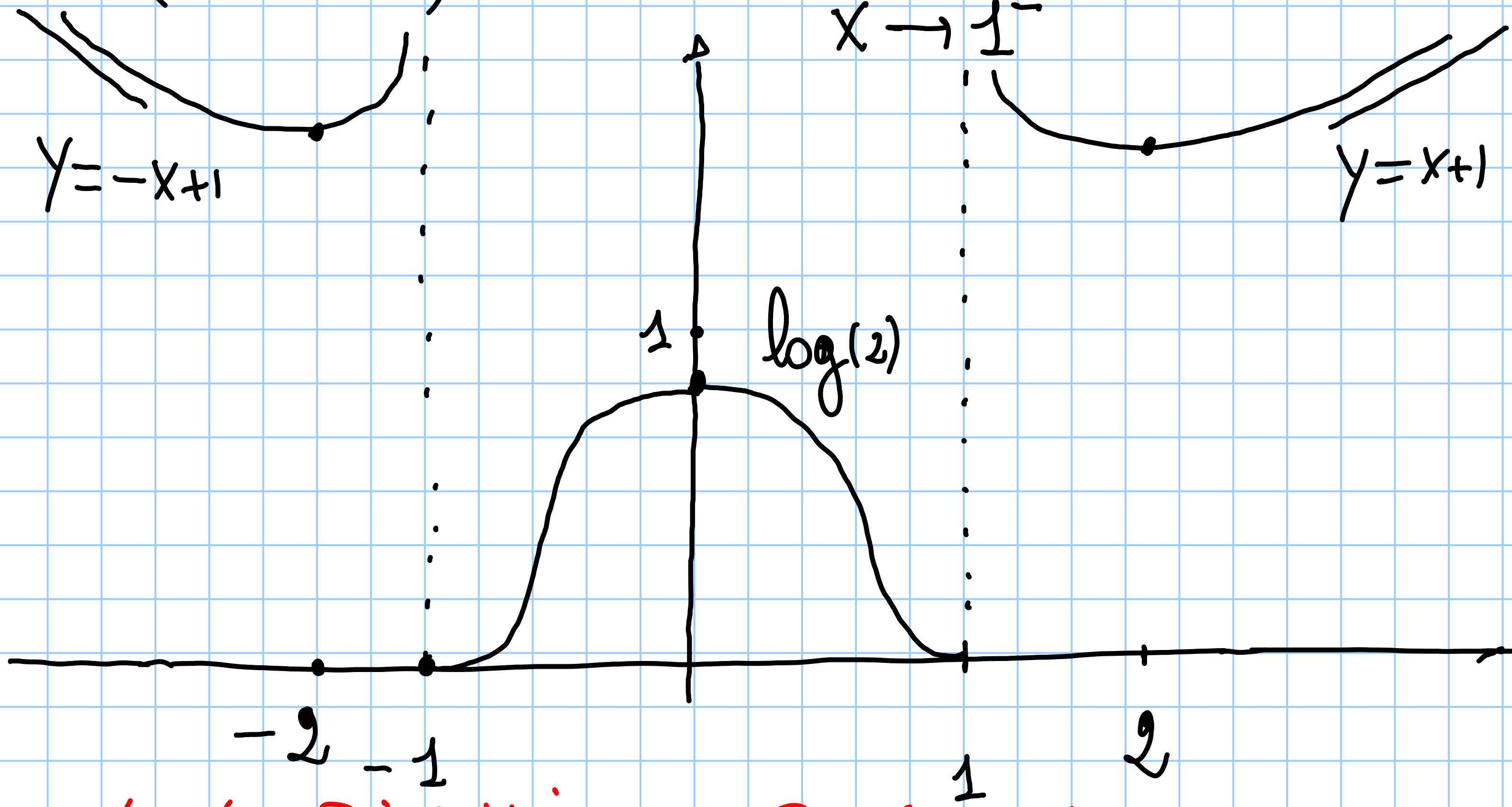
$$f'(x) = \frac{e^{\overbrace{\frac{1+o(1)}{x-1}}^{\to -\infty,\ x\to 1^-}}\,(-1+o(1))}{\underbrace{(x-1)^2}_{\to 0,\ x\to 1^-}} =$$

$\to -\infty,\ x\to 1^-$, $e^{-\infty} = 0$

$$(-1+o(1))\, e^{\frac{1+o(1)}{x-1} - \log(x-1)^2} =$$

$$= (-1+o(1))\, e^{\frac{1}{x-1}} \left(1+o(1) - (x-1)\log(x-1)^2\right)$$

$$= (-1+o(1))\, e^{\frac{1+o(1)}{x-1}} \xrightarrow[x\to 1^-]{} (-1)\, e^{-\infty} = 0^-$$



NON RICHIESTO NEL TESTO

$$f(x) = \log\left(1+e^{x\frac{1}{1-x^{-1}}}\right) = \{x\to+\infty\}$$

$$= \log\left(1+e^{x\left(1+\frac{1}{x}+\frac{1}{x^2}+o\left(\frac{1}{x^2}\right)\right)}\right) =$$

$$= \log\left(e^{x+1+\frac{1}{x}+o\left(\frac{1}{x}\right)}\right) +$$

$$\log\left(1 + e^{-x+o(x)}\right) =$$

$$x + 1 + \frac{1}{x} + o\left(\frac{1}{x}\right),$$

$$f(x) > x+1, \quad x \longrightarrow +\infty$$

- $$1 + e^{\frac{x^2}{|x|-1}} > 1, \ \forall x \in \mathrm{dom}(f)$$

$$\log\left(1 + e^{\frac{x^2}{|x|-1}}\right) > \log(1) = 0$$

$$\int_1^{+\infty} \underbrace{\frac{2\,\mathrm{arcTg}(x-1)}{(x+1)^{\alpha}}}_{f_\alpha(x)}\,dx$$

DISCUTERE CONVERGENZA,

CALCOLO PER $\alpha = 2$

$$f_\alpha \in \mathcal{R}(1,t) \quad \forall\, t > 1$$

$$f_\alpha(x) \underset{x\to+\infty}{=} 2 \cdot \frac{\frac{\pi}{2} + o(1)}{(x+1)^{\alpha}} \,,$$

T. CONFRONTO ASINTOTICO

CONVERGE $\iff \alpha > 1$

$$I = \int_1^{+\infty} \frac{2\,\mathrm{arctg}\,(x-1)}{(x+1)^2}\,dx \qquad \boxed{\alpha = 2}$$

$$\int_1^{+\infty} \frac{2\,\mathrm{arctg}\,(x-1)}{(x+1)^2}\,dx = \lim_{M \to +\infty} \int_1^{M} \frac{2\,\mathrm{arctg}\,(x-1)}{(x+1)^2}\,dx$$

$$\int_1^{M} 2\,\frac{\mathrm{arctg}\,(x-1)}{(x+1)^2}\,dx = \int_1^{M} 2\left(-\frac{1}{(x+1)}\right)'\mathrm{arctg}(x-1)\,dx$$

$$= \underbrace{-2\,\frac{\operatorname{arcTg}(x-1)}{x+1}\Bigg|_1^M}_{\substack{\downarrow\ M\to+\infty \\ 0}} + \int_1^M \frac{2}{(x+1)\left(1+(x-1)^2\right)}\,dx$$

$$\frac{2}{(x+1)\left(1+(x-1)^2\right)} = \frac{A}{x+1} + \frac{2B(x-1)+C}{1+(x-1)^2}$$

$$A + A(x-1)^2 + 2B(x^2-1) + C(x+1) = 2$$

$$A + Ax^2 - 2Ax + A + 2Bx^2 - 2B + Cx + C$$

$$\begin{cases} 2A - 2B + C = 2 \\ -2A + C = 0 \\ A + 2B = 0 \end{cases} \qquad \begin{cases} 2A + A + 2A = 2 \\ C = 2A \\ 2B = -A \end{cases}$$

$$\begin{cases} A = 2/5 \\ C = 4/5 \\ B = -A/2 = -1/5 \end{cases}$$

$$= \int_1^M \frac{2}{5} \frac{dx}{x+1} + \int_1^M -\frac{2}{5} \frac{x-1}{1+(x-1)^2} dx +$$

$$+ \int_1^M 4/5 \frac{dx}{1+(x-1)^2} =$$

$$\frac{2}{5}\left(\log(1+M) - \log(2)\right)$$

$$-\frac{1}{5} \log\left(1+(x-1)^2\right)\Big|_1^M + \frac{4}{5} \operatorname{arcTg}(M-1) =$$

$$\frac{2}{5} \log(1+M) + \frac{4}{5} \operatorname{arcTg}(M-1)$$

$$-\frac{1}{5}\log\left(1+(M-1)^2\right)-\frac{2}{5}\log(2) =$$

$$\frac{1}{5}\log\boxed{\left(\frac{(1+M)^2}{1+(M-1)^2}\right)} + \frac{4}{5}\left(\frac{\pi}{2}+o(1)\right)$$

$\to 1,\ M \to +\infty$

$$-\log(2)$$

$$= \frac{2}{5}\pi - \frac{2}{5}\log(2) + o(1),\ M \to +\infty$$

$$I = \frac{2}{5}\left(\pi - \log(2)\right)$$