$$z = \rho e^{i\theta} = \rho(\cos(\theta) + i\sin(\theta))$$

$$= \rho e^{i(\theta + 2k\pi)}, \quad k \in \mathbb{Z}$$

$$z_1 = \rho_1 e^{i\theta_1}, \quad z_2 = \rho_2 e^{i\theta_2}$$

$$= \rho_1 \cos\theta_1 + i\rho_1 \sin\theta_1 \qquad = \rho_2 \cos\theta_2 + i\rho_2 \sin\theta_2$$

$$z_1 \cdot z_2 = \rho_1\rho_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) +$$

$$i\rho_1\rho_2(\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1)$$

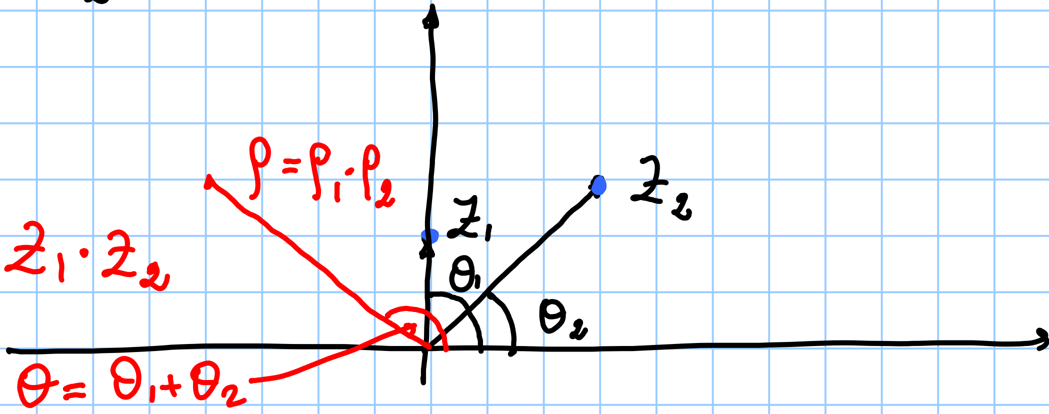
$$\left\{ = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1) \right\}$$

$$= \rho_1\rho_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

$$= \rho_1\rho_2 e^{i(\theta_1+\theta_2)}$$

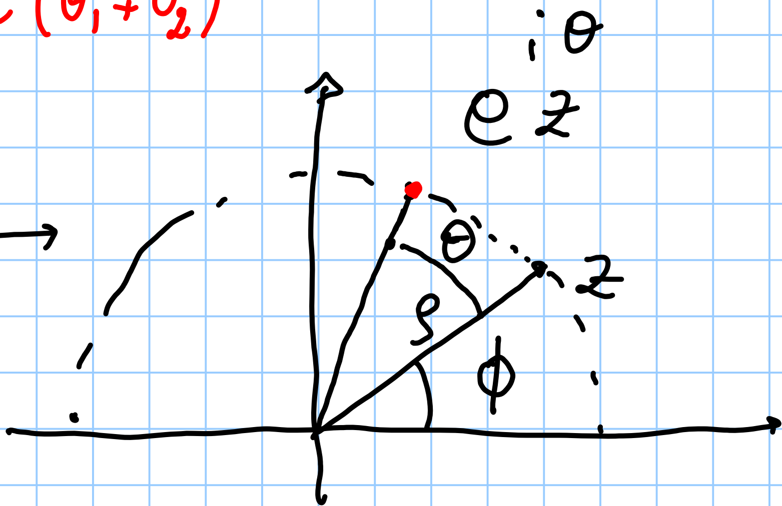
$z_1 = e^{i\pi/2}$ $\qquad$ $z_2 = 2e^{i\frac{\pi}{4}}$

$$z_1 \cdot z_2 = 2e^{i\left(\frac{\pi}{2}+\frac{\pi}{4}\right)} = 2e^{i\frac{3}{4}\pi}$$



$$z_1 \cdot z_2 = \rho_1 \rho_2 e^{i(\theta_1+\theta_2)}$$

$z = \rho e^{i\phi} \to e^{i\theta} z \to$



$\theta > 0$

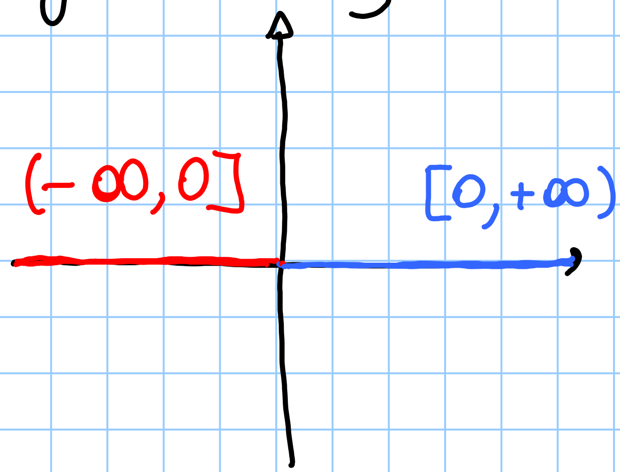
$e^{i\theta} z$ SI OTTIENE

RUOTANDO z DI UN ANGOLO θ

IN SENSO ANTIORARIO

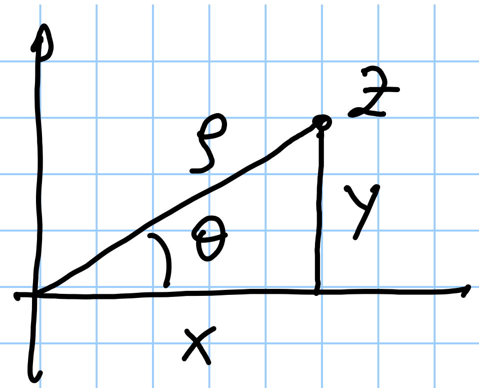
NOTA: $[0,+\infty) = \{ z \in \mathbb{C} : \arg(z) = 0 \}$

$(-\infty,0] = \{ z \in \mathbb{C} : \arg(z) = \pi \}$



$$(\rho,\theta) \longrightarrow (x,y) \qquad \begin{cases} x = \rho\cos(\theta) \\ y = \rho\sin(\theta) \end{cases}$$

$$(x,y) \longrightarrow (\rho,\theta) \qquad ?$$



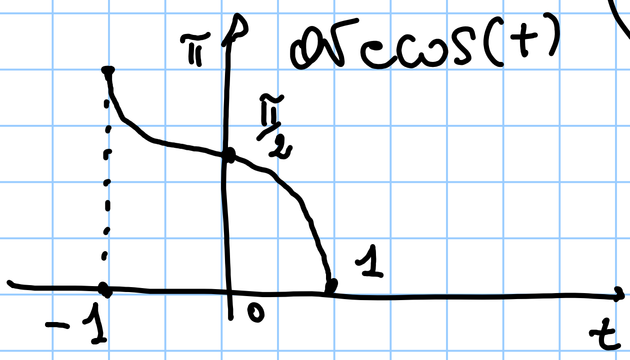
$$\rho = \sqrt{x^2+y^2}$$

$$|z| := \sqrt{x^2+y^2}$$

MODULO di z, $|z|$

$$x = \rho\cos(\theta) \qquad \cos(\theta) = \frac{x}{\rho}$$

$$\text{"}\theta = \arccos\left(\frac{x}{\rho}\right)\text{"}, \quad \frac{x}{\rho} \in [-1,1]$$

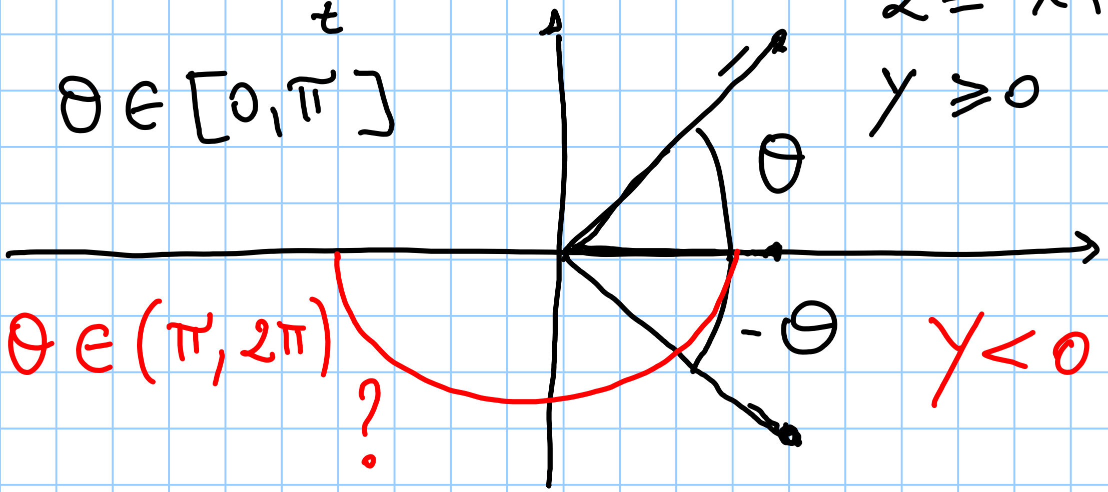


$\longrightarrow \theta \in [0,\pi]$

$\theta \in [0,\pi]$

$z = x+iy$

$y \geq 0$



$\theta \in (\pi, 2\pi)$?

$y<0$?

$$\theta = \begin{cases} \arccos\left(\frac{x}{\rho}\right), & x \geq 0 \\ -\arccos\left(\frac{x}{\rho}\right), & y < 0 \end{cases} \quad \begin{array}{|l} [0,\pi] \\ (-\pi,0) \end{array}$$

$$\longrightarrow \quad \theta \in (-\pi, \pi]$$

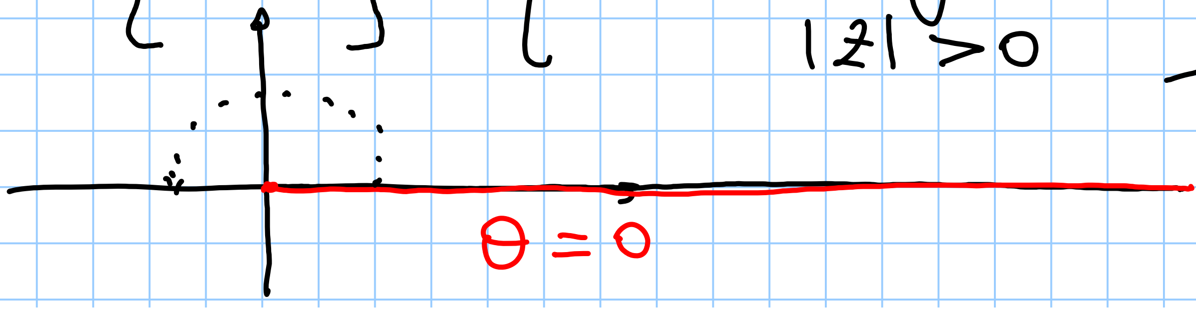
$$\theta \in [0, 2\pi)$$

$$\theta = \begin{cases} \arccos\left(\frac{x}{\rho}\right), & y \geq 0 \\ -\arccos\left(\frac{x}{\rho}\right) + 2\pi, & y < 0 \end{cases} \quad \begin{array}{|l} \theta \in [0,\pi] \\ \theta \in (\pi, 2\pi) \end{array}$$

$$z = 2, \quad |z| = 2, \quad \begin{cases} \theta = \arccos\left(\frac{2}{2}\right) = 0 \\ y = 0 \end{cases}$$

$$z = 2e^{i\theta} = 2e^{i0} = 2$$

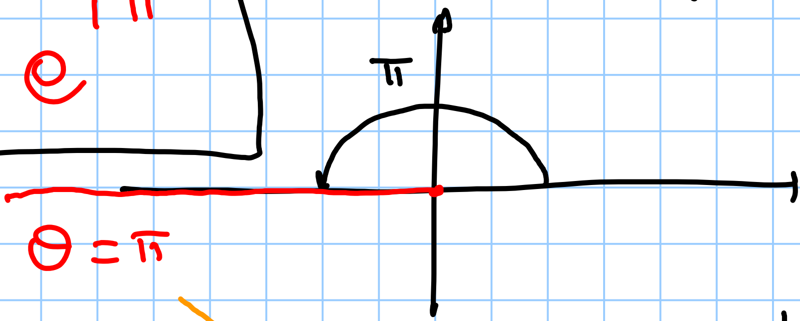
$$(0, +\infty) = \mathbb{R} \cap \{x > 0\} = \left\{ z \in \mathbb{C} : \begin{matrix} \arg(z) = 0 \\ |z| > 0 \end{matrix} \right\}$$

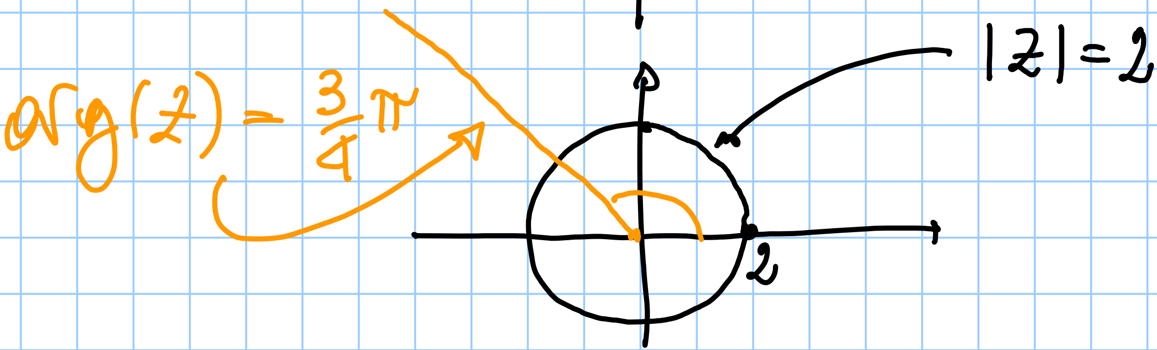


$z=-2$, $|z|=2$ $\qquad \begin{cases}\theta=\arccos\left(-\frac{2}{2}\right)=\pi\\ y=0\end{cases}$

$z=2e^{i\pi}$ $\qquad (-\infty,0)=\left\{z\in\mathbb{C}:\arg(z)=\pi,\ |z|>0\right\}$

$$\boxed{-1=e^{i\pi}}$$





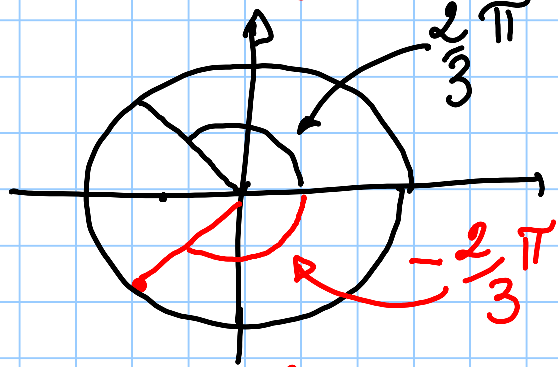
$z=1+i\sqrt{3}$, $|z|=\sqrt{1+3}=2$

$$\begin{cases}\theta=\arccos\left(\frac{1}{2}\right)=\frac{\pi}{3}\\ y=\sqrt{3}>0\end{cases}$$

$z=2e^{i\pi/3}$

$z = -1 - i\sqrt{3}$, $|z| = \sqrt{1+3} = 2$

$$\begin{cases} \theta = -\arccos\left(-\frac{1}{2}\right) = -\frac{2}{3}\pi \in (-\pi, 0) \\ y = -\sqrt{3} < 0 \end{cases}$$



$$\theta = -\frac{2}{3}\pi + 2\pi = \frac{4}{3}\pi$$

$$z = 2e^{-i\frac{2}{3}\pi} = 2e^{i\frac{4}{3}\pi}$$

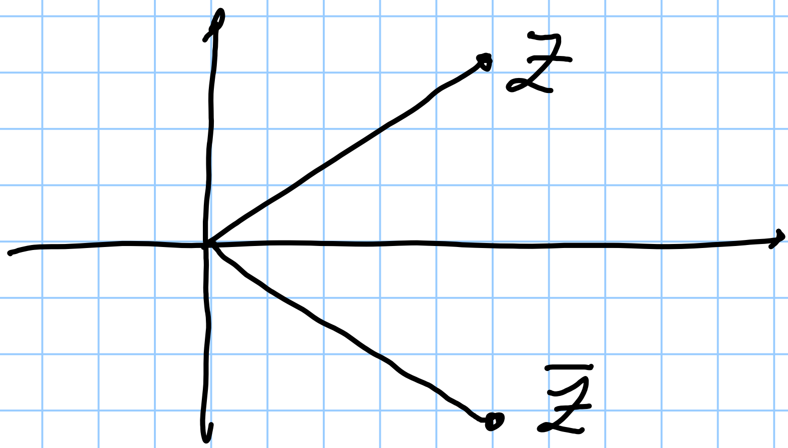
$\boxed{z = 0} \longrightarrow \theta$ NON E' DEFINITO

$z = 0 \longrightarrow \rho = 0$, $\boxed{\theta \in \mathbb{R}}$

z_1 / z_2 ?

IL COMPLESSO CONIUGATO di z,

$$z = x + iy \longrightarrow \bar{z} = x - iy$$



$$\begin{cases} z + \bar{z} = 2x \\ z - \bar{z} = 2iy \end{cases} \qquad \begin{cases} x = \dfrac{z + \bar{z}}{2} \\ y = \dfrac{z - \bar{z}}{2i} \end{cases}$$

$$|z| = \sqrt{x^2 + y^2} = \rho \quad ;$$

$$|z|^2 = x^2 + y^2$$

$$z \cdot \bar{z} = x^2 + y^2 + i(xy - yx) = x^2 + y^2 =$$
$$= |z|^2$$

$$z \cdot \bar{z} = x^2 + y^2 \quad , \quad |z| = \sqrt{z \cdot \bar{z}}$$

$$z = |z| e^{i\theta} = |z| \left(\cos(\theta) + i \sin(\theta) \right)$$

$$\bar{z} = |z| e^{-i\theta} = |z| \left(\cos(\theta) - i \sin(\theta) \right)$$

$$z \cdot \bar{z} = |z| e^{i\theta} \, |z| e^{-i\theta} = |z|^2 e^{i\theta - i\theta}$$
$$= |z|^2$$

DIVISIONE

$$z_1 \in \mathbb{C} \quad , \quad z_2 \in \mathbb{C} \setminus \{0\}$$

$$\frac{z_1}{z_2} := \frac{z_1 \, \overline{z_2}}{z_2 \cdot \overline{z_2}} = \frac{z_1 \cdot \overline{z_2}}{|z_2|^2}$$

$$z = x + iy \; ; \qquad \frac{z}{i} = \frac{z\,(-i)}{i(-i)} = -iz \; ,$$

$$z = \frac{1+i\sqrt{2}}{3-i} = \frac{(1+i\sqrt{2})(3+i)}{|3-i|^2} =$$

$$= \frac{3-\sqrt{2}+i(3\sqrt{2}+1)}{3^2+1^2} = \frac{3-\sqrt{2}}{10} + i\,\frac{3\sqrt{2}+1}{10}$$

$$z_1 = \rho_1 e^{i\theta_1} \qquad z_2 = \rho_2 e^{i\theta_2}$$

$$\frac{z_1}{z_2} = \frac{z_1 \cdot \overline{z_2}}{|z_2|^2} = \frac{\rho_1}{\rho_2} e^{i(\theta_1-\theta_2)}$$

$$= \frac{\rho_1}{\rho_2}\left(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right)$$

$$\frac{1}{i} = \frac{1}{e^{i\pi/2}} = e^{-i\pi/2} \quad , \quad \frac{1}{\left(e^{i\frac{\pi}{2}}\right)} = \left(e^{i\frac{\pi}{2}}\right)^{-1} = e^{-i\frac{\pi}{2}}$$

$$\frac{z}{i} = \frac{z}{e^{i\pi/2}} = e^{-i\frac{\pi}{2}} z$$

$$\frac{z}{i} = \frac{z}{e^{i\pi/2}} = e^{-i\frac{\pi}{2}} z = e^{i\frac{3}{2}\pi} z = e^{i\pi + i\frac{\pi}{2}} z$$

$$= e^{i\pi} e^{i\frac{\pi}{2}} z = -iz$$

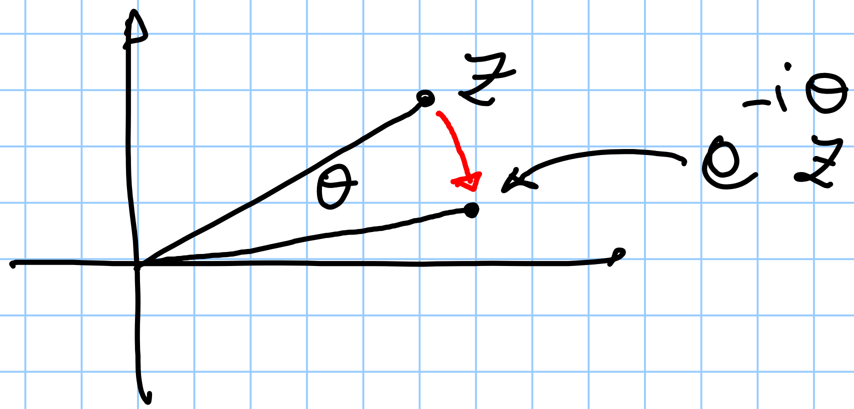
$$z_1 = 2e^{i\frac{\pi}{4}}, \qquad z_2 = 3\, e^{-i\frac{\pi}{2}}$$

$$\frac{z_1}{z_2} = \frac{2}{3} e^{i\left(\frac{\pi}{4} - \left(-\frac{\pi}{2}\right)\right)} = \frac{2}{3} e^{i\left(\frac{\pi}{4} + \frac{\pi}{2}\right)} = $$

$$= \frac{2}{3} e^{i\frac{3}{4}\pi}$$

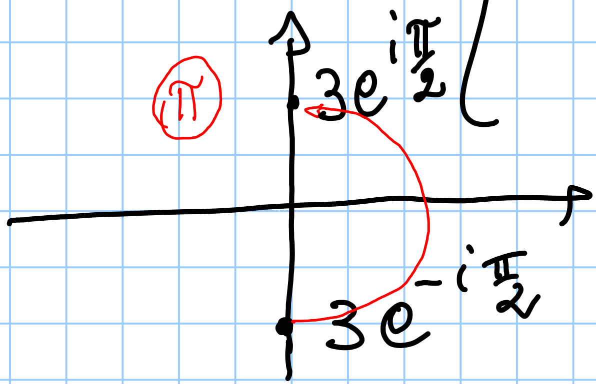
$\theta > 0$

$$z \longrightarrow \frac{z}{e^{i\theta}} = e^{-i\theta} z$$



$$\begin{cases} z = 2e^{i\frac{\pi}{4}} \\ \quad = \rho e^{i\theta} \end{cases}$$

$$\begin{cases} z = -3e^{-i\frac{\pi}{2}} = \left\{ -1 = e^{i\pi} \right\} = e^{i\pi} 3 e^{-i\frac{\pi}{2}} = 3e^{i(\pi - \frac{\pi}{2})} = 3e^{i\frac{\pi}{2}} \\ \quad = \rho e^{i\theta} \end{cases}$$



z^m, $m \in \mathbb{N}$, $z = |z| e^{i\theta}$

$$z^m = \left(|z| e^{i\theta} \right)^m = |z|^m e^{i\theta m}$$

$$z = \sqrt{3} + i \quad , \quad |z| = 2 \quad , \quad y > 0$$

$$\vartheta = \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$z = 2e^{i\pi/6}$$

$$z^3 = 2^3 e^{i3\pi/6} = 2^3 e^{i\frac{\pi}{2}} = i8$$

$$z^5 = 2^5 e^{i5\cdot\frac{\pi}{6}} = 2^5\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = 2^4\left(-\sqrt{3} + i\right)$$

$$z^{30} = 2^{30} e^{i30\frac{\pi}{6}} = 2^{30} e^{i5\pi} =$$

multiplo di 2π

$$= 2^{30} e^{i(\pi + 4\pi)} = 2^{30} e^{i\pi}$$

$$= -2^{30}$$

multiplo di 6

$$z^{35} = 2^{35} e^{i35/6\,\pi} = 2^{35} e^{i\frac{30+5}{6}\pi} =$$

$$= 2^{35} e^{i\left(5\pi + \frac{5}{6}\pi\right)} =$$

$$2^{35} e^{i5\pi} e^{i\frac{5\pi}{6}} =$$

$$= 2^{35} e^{i\pi} e^{i\frac{5}{6}\pi} = 2^{35} e^{i\frac{11}{6}\pi} = 2^{35}\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$$

$$= 2^{34}\left(\sqrt{3} - i\right)$$

$z \in \mathbb{C}$, $m \in \mathbb{N}$

RADICE m-esima di z:

qualunque $w \in \mathbb{C}$: $w^m = z$

$$z = |z| e^{i\theta} = |z| e^{i(\theta + 2k\pi)} \quad k \in \mathbb{Z}$$

$$w = |w| e^{i\phi}, \quad w^m = |w|^m e^{im\phi}$$

$$w^m = z \iff |w|^m e^{im\phi} = |z| e^{i(\theta + 2\pi k)} \quad k \in \mathbb{Z}$$

$$|w|^m = |z|, \quad m\phi = \theta + 2k\pi, \; k \in \mathbb{Z}$$

$$|W| = \sqrt[m]{|z|} \qquad \phi = \phi_k = \frac{\theta}{m} + \frac{2\pi k}{m}, \; k \in \mathbb{Z}$$

$$W = W_k = \sqrt[m]{|z|}\, e^{i\phi_k}, \; k \in \mathbb{Z}$$

$m = 2$, $z = -1$

$W^2 = -1$, $z = -1 = e^{i\pi}$, $\theta = \pi$

$$\phi_k = \frac{\pi}{2} + \frac{2\pi k}{2} =$$

$$|W| = \sqrt{|-1|} = 1$$

$$= \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}$$

$$W_k = \sqrt{|-1|}\, e^{i\phi_k} =$$

$$= e^{i\pi/2} e^{ik\pi} = i\, e^{ik\pi}, \; k \in \mathbb{Z}$$

$$W_k = i \cdot e^{ik\pi} = i \cdot \begin{cases} 1, & k \in \{\ldots, -4, -2, 0, 2, 4, \ldots\} \\ -1, & k \in \{\ldots, -5, -3, -1, 1, 3, 5 \ldots\} \end{cases}$$

$$\boxed{W_k = i\, e^{ik\pi}, \; k = 0, 1}$$

LE RADICI QUADRATE $(n=2)$ di -1,

CIOE' LE SOLUZIONI (in $\mathbb{C}$) di $W^2 = -1$

SONO $W_0 = i$ e
$W_1 = -i$

$$W_0 = i\, e^{i 0 \cdot \pi} = i \cdot 1 = i$$

$$W_1 = i\, e^{i 1 \cdot \pi} = i\, e^{i\pi} = -i$$

$$W^2 + 1 = 0 \iff W = \pm i$$

CALCOLIAMO LE RADICI CUBICHE
$(n=3)$ di $z=1$

$W^3=1$ (NOTA: se $W=x\in\mathbb{R}$ $x^3=1 \Leftrightarrow x=1$)

$$\boxed{z=1} \longrightarrow \boxed{W^3=1}$$

$$z=e^{i0} \longrightarrow W_k=\sqrt[3]{|z|}\,e^{i\phi_k}$$

$|z|=1$, $\phi_k=\frac{0}{3}+\frac{2k\pi}{3}$, $k\in\mathbb{Z}$

$$W_k=e^{i\frac{2\pi}{3}k}=\begin{cases}1, & k\in\{\ldots,0,3,6,9\ldots\}\\ e^{i\frac{2}{3}\pi}, & k\in\{\ldots,1,4,7,\ldots\}\\ e^{i\frac{4}{3}\pi}, & k\in\{\ldots,2,5,8,\ldots\}\end{cases}$$

$$W_k=e^{i\frac{2\pi}{3}k},\ k=0,1,2,\quad \begin{cases}W_0=1\\ W_1=e^{i\frac{2}{3}\pi}\\ W_2=e^{i\frac{4}{3}\pi}\end{cases}$$

$$W \in \left\{ 1, e^{i\frac{2}{3}\pi}, e^{i\frac{4}{3}\pi} \right\} = \left\{ W_k \right\}_{k=0,1,2}$$

$$z = \rho e^{i\theta}, \qquad W^m = z$$

$$|W| = \sqrt[m]{|z|} \qquad \phi_k = \frac{\theta}{m} + \frac{2\pi k}{m}, \quad k \in \mathbb{Z}$$

$$\begin{cases} \phi_0 = \theta/m \overset{\text{mod } 2\pi}{=} \underset{k=m}{\theta/m + 2\pi} \overset{\text{mod } 2\pi}{=} \underset{k=2m}{\theta/m + 4\pi} = \dots \\ W_0 = \sqrt[m]{|z|}\, e^{i\phi_0}, \quad k = 0 + \text{multiplo di } m \end{cases}$$

$$\begin{cases} \phi_1 = \theta/m + \frac{2\pi}{m} \overset{\text{mod } 2\pi}{=} \theta/m + \frac{2\pi}{m} + 2\pi = \frac{\theta}{m} + \frac{2\pi}{m}(1+m) \\ \qquad \overset{\text{mod } 2\pi}{=} \theta/m + \frac{2\pi}{m}(1+m) + 2\pi = \\ \qquad = \theta/m + \frac{2\pi}{m}(1+2m) = \dots \\ W_1 = \sqrt[m]{|z|}\, e^{i\phi_1}, \quad k = 1 + \text{multiplo di } m \end{cases}$$

$$\begin{cases} \phi_2 \overset{\text{mod } 2\pi}{=} \theta/m + \dfrac{2\pi}{m}(2+\ell m), \quad \ell \in \mathbb{Z} \\ \qquad k = 2 + \text{multiplo di } m \\ W_2 = \sqrt[m]{|z|}\, e^{i\phi_2}, \quad k = 2 + \text{multiplo di } m \end{cases}$$

$$\vdots$$

$$\begin{cases} \phi_{m-1} \overset{\text{mod } 2\pi}{=} \theta/m + \dfrac{2\pi}{m}(m-1+\ell m), \quad \ell \in \mathbb{Z} \\ \qquad k = m-1 + \text{multiplo di } m \\ W_{m-1} = \sqrt[m]{|z|}\, e^{i\phi_{m-1}}, \quad k = m-1 + \text{multiplo di } m \end{cases}$$

$$\phi_m \overset{\text{mod } 2\pi}{=} \theta/m + \frac{2\pi}{m}(m+\ell m) =$$

$k = m + \text{multiplo di } m,$

$$= \theta/m + \frac{2\pi}{m}(0 + (1+\ell)m) =$$

$$= \theta/m + \frac{2\pi}{m}(0 + m), \quad m \text{ multiplo di } m.$$

$$\underset{\mathrm{mod}\, 2\pi}{=} \phi_0 \quad !$$

$$z = |z| e^{i\theta}, \quad m \in \mathbb{N}$$

TUTTE LE SOLUZIONI DISTINTE di $W^m = z$ SONO

$$W_k = \sqrt[m]{|z|}\, e^{i\left(\frac{\theta}{m} + \frac{2\pi}{m}k\right)}, \quad k = 0, 1, 2, \dots, m-1$$

CALCOLIAMO LE RADICI m-esime di 1

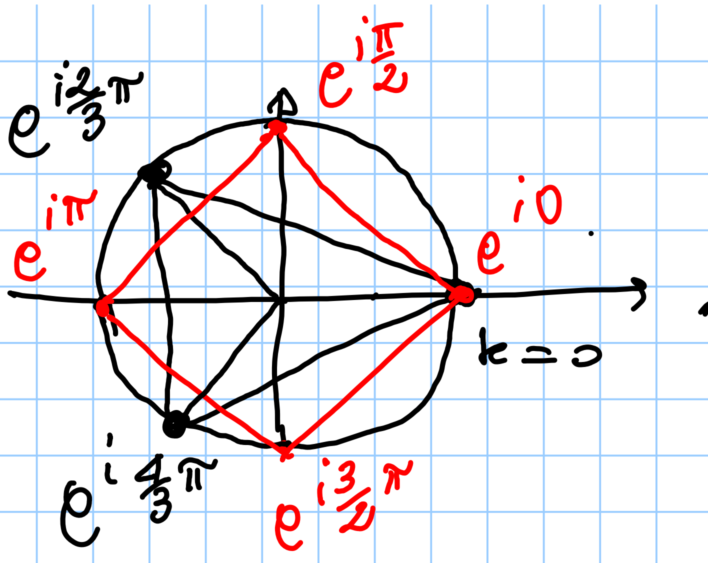
$$W_k = e^{i\frac{2\pi}{m}k}, \quad k = 0, 1, \dots, m-1$$

RADICI QUADRATE di $z = 1$, $W^2 = 1$

$$W_k = e^{ik\pi}, \quad k = 0, 1 \quad \Big| \quad W^2 - 1 = 0, \quad W_\pm = \pm 1$$

$$W^3 = 1$$

$$W_k = e^{i\frac{2\pi}{3}k}, \quad k = 0, 1, 2.$$



$$W^4 = 1$$

$$W_k = e^{i\frac{2\pi}{4}k}, \quad k = 0, 1, 2, 3. \quad \left\{1, e^{i\frac{\pi}{2}}, e^{i\pi}, e^{i\frac{3}{2}\pi}\right\}$$

W_k, $k = 0, 1, .., m-1$: $W^m = 1$

SONO i VERTICI del POLIGONO REGOLARE INSCRITTO di m LATI

$$\xi_k^{(m)} = e^{i\frac{2\pi}{m}k}, \quad k = 0, 1, 2, .., m-1$$

USANDO LE $\xi_k^{(m)}$ si POSSONO RAPPRESENTARE

FACILMENTE LE RADICI m-esime di un numero COMPLESSO, $z \in \mathbb{C}$

$z = |z| e^{i\theta}$ $\qquad W^m = z$

$$W_k = \underbrace{\sqrt[m]{|z|}\, e^{i\frac{\theta}{m}}}_{W_0} \xi_k^{(m)}, k = 0, 1, 2, \dots, m-1$$

$z = 1$, $\quad W^2 = 1$, $\quad \begin{cases} \xi_0^{(2)} = 1 \\ \xi_1^{(2)} = -1 \end{cases}$

$W_k = \xi_k^{(2)}$, $k = 0, 1$

$W_0 = 1$, $\quad W_1 = -1$

$z = -1$, $\quad W^2 = -1$

$W_k = e^{i\frac{\pi}{2}} \xi_k^{(2)}$, $k = 0, 1$

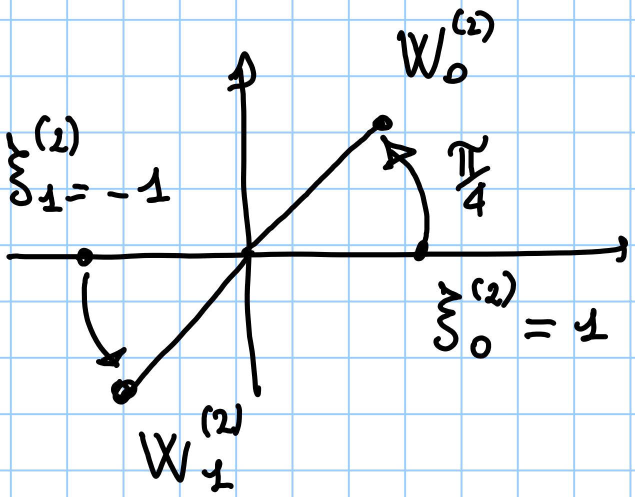
$W_0 = e^{i\frac{\pi}{2}} = i$, $\quad W_1 = -e^{i\frac{\pi}{2}} = -i$

$$z = i = e^{i\frac{\pi}{2}} \quad , \quad W^2 = i$$

$$W_k = e^{i\frac{\pi}{4}} \, \xi_k^{(2)} \quad , \quad k = 0, 1$$

$$W_0 = e^{i\frac{\pi}{4}} \quad , \quad W_1 = -e^{i\pi/4}$$

IN PARTICOLARE L'INSIEME



$$\left\{ W_k \right\}_{k=0,1,2,\dots,m-1}$$

$$W_k = e^{i\theta/m} \, \xi_k^{(m)}$$

$$k = 0, 1, 2, \dots m-1$$

SI OTTIENE RUOTANDO

$$\left\{ \xi_k^{(m)} \right\}_{k=0,1,2,\dots m-1}$$

di un angolo θ/m in SENSO ANTIORARIO