NUMERI COMPLESSI

$$x^2 - 1 = 0 \quad , \quad x_{\pm} = \pm 1$$

$$(x^2 - 1) = (x-1)(x+1)$$

$$(x-1)^2 = 0 \quad , \quad x_{\pm} = 1$$

$$(x-1)^2 = (x-1)(x-1)$$

$$x^2 + 1 = 0 \qquad x_{\pm} \ ?$$

$$\boxed{x^2 = -1} \Rightarrow x \notin \mathbb{R}$$

UNITÀ IMMAGINARIA: i

$$\boxed{i^2 = -1}$$

" " " $i = \sqrt{-1}$ " " "

DA USARE CON PRUDENZA

$$i^2 = i\, i = \sqrt{-1}\,\sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$$

???

$x^2 = -1 \qquad x_\pm = \pm i$

$(-1)\cdot i = -i\,;$

IL PRODOTTO DEVE ESSERE ASSOCIATIVO

$$(-i)^2 = (-i)(-i) = (-1)(-1)(i)(i) = (i)^2 = -1$$

$$(x^2+1) = (x-i)(x+i)$$

VALGONO LE LEGGI DISTRIBUTIVE

$$(x-i)(x+i) = x^2 - ix + ix - i^2 = x^2+1$$

NUMERI COMPLESSI

$$\mathbb{C} = \left\{ z = x+iy,\ x\in\mathbb{R},\ y\in\mathbb{R} \right\}$$

$\boxed{i^2 = -1}$ i UNITA' IMMAGINARIA

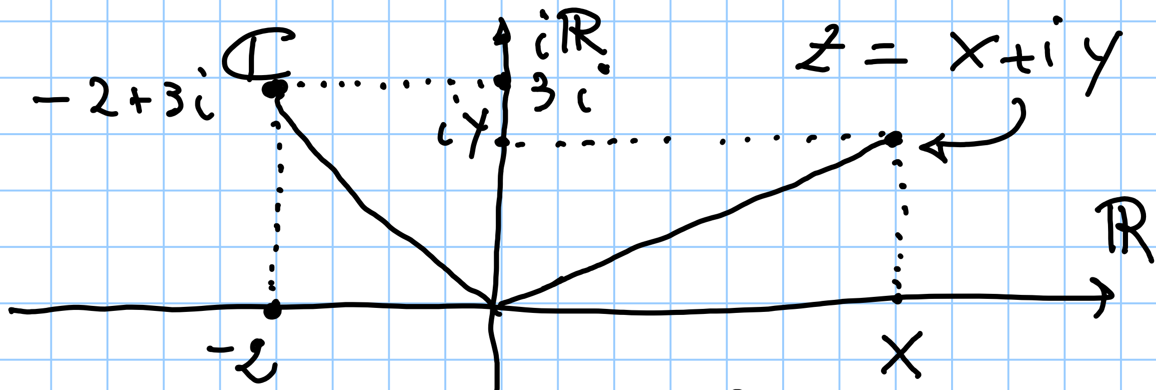
$x = Re(z)$ PARTE REALE di z

$y = Im(z)$ PARTE IMMAGINARIA di z

$\mathbb{R} \subset \mathbb{C}$, $\mathbb{R} = \{ z \in \mathbb{C} : Im(z) = 0 \}$

$i\mathbb{R} \subset \mathbb{C}$, $i\mathbb{R} = \{ z \in \mathbb{C} : Re(z) = 0 \}$

NUMERI IMMAGINARI PURI



$z = -2$

$\mathbb{R} = \{ z = x+iy : y = 0 \}$

$z = 3i$ $\quad z = -2+3i$

$i\mathbb{R} = \{ z = x+iy : x = 0 \}$

$$z_1 = x_1 + i y_1 \quad , \quad z_2 = x_2 + i y_2$$

$$z_1 + z_2 = (x_1 + i y_1) + (x_2 + i y_2)$$

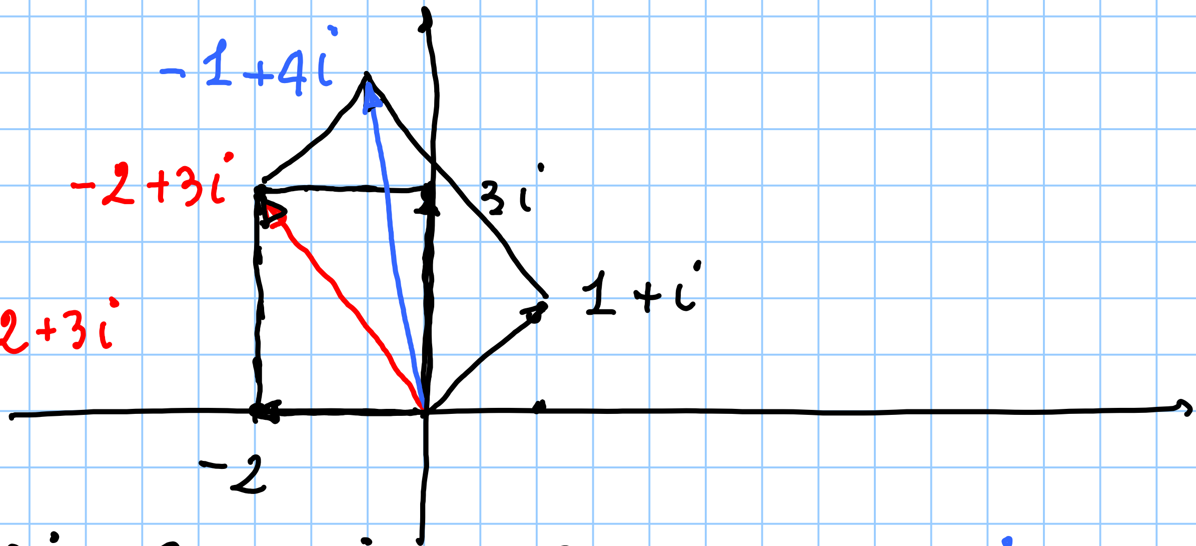
$$:= (x_1 + x_2) + i (y_1 + y_2)$$

SOMMA in $\mathbb{C}$

$z_1 = -2$

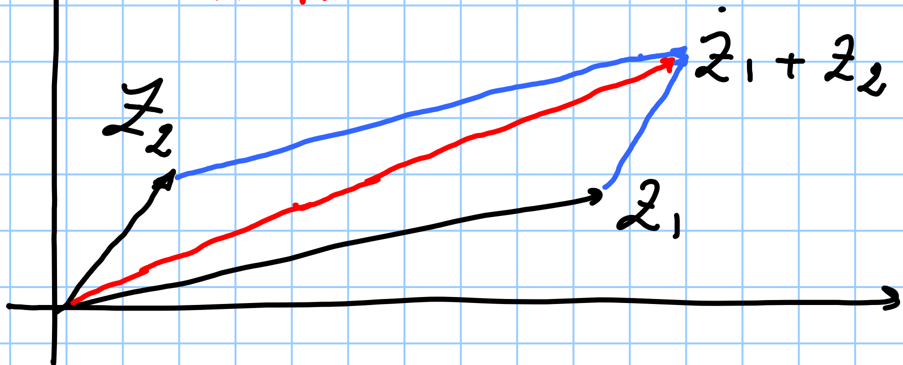
$z_2 = 3i$

$z_1 + z_2 = -2 + 3i$



$z_1 = -2 + 3i \ , \ z_2 = 1 + i \qquad z_1 + z_2 = -1 + 4i$

REGOLA del PARALLELOGRAMMA



$\forall z \in \mathbb{C} \quad \exists$ UNUNICO $w \in \mathbb{C}$

ELEMENTO OPPOSTO RISPETTO ALLA SOMMA

$$w + z = 0$$

$$w := -z \;;$$

$$z = x + iy \Rightarrow -z = -x + i(-y) = -x - iy$$

$$-z \equiv (-1) \cdot z = (-1)(x+iy)$$
$$= -x - iy .$$

OVVERO L'OPPOSTO di z è

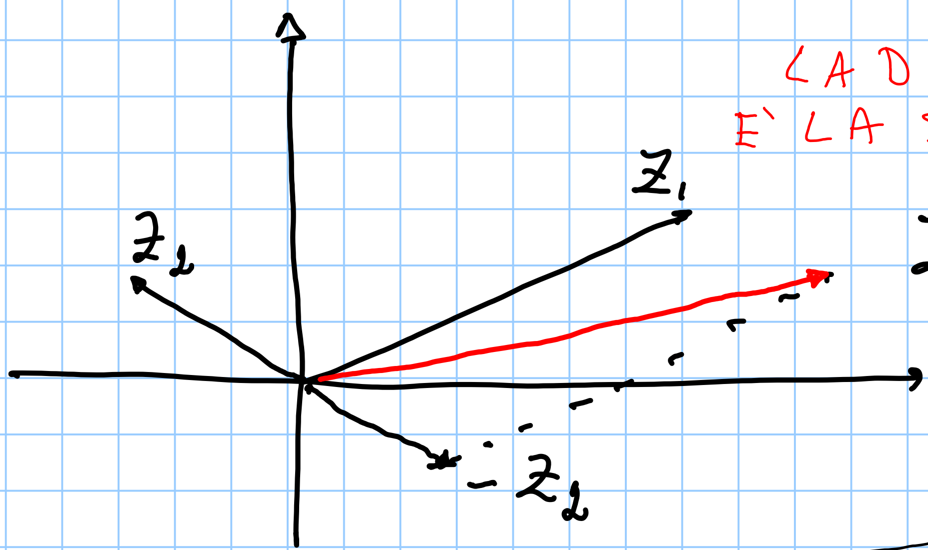
L'OPPOSTO di 1 PER z.

DIFFERENZA

$$z_1 - z_2 \,; \quad z_1 = x_1 + i y_1$$
$$z_2 = x_2 + i y_2$$

$$-z_2 = -x_2 - i y_2$$

$$z_1 - z_2 := z_1 + (-z_2) =$$
$$(x_1 - x_2) + i(y_1 - y_2)$$



LA DIFFERENZA $z_1 - z_2$ È LA SOMMA di z_1 e $-z_2$.

$$z_1 - z_2 = z_1 + (-z_2)$$

PRODOTTO

$$z_1 \cdot z_2 = (x_1 + i y_1)(x_2 + i y_2) =$$

$$i^2 = -1$$

LEGGE DISTRIBUTIVA

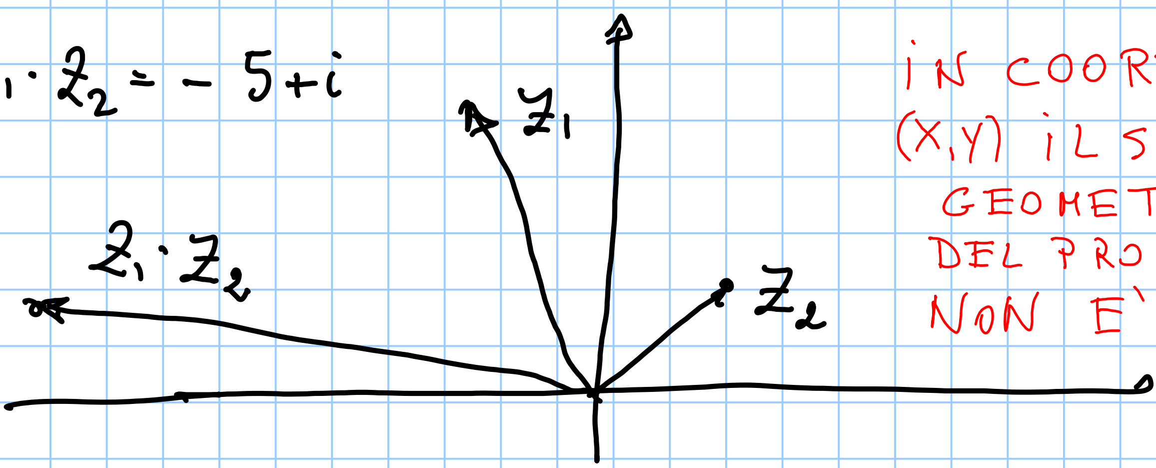
$$= x_1 x_2 + i y_1 x_2 + i x_1 y_2 + (i)^2 y_1 y_2$$

$$z_1 \cdot z_2 := (x_1 x_2 - y_1 y_2) + i\,(x_1 y_2 + x_2 y_1)$$

$$z_1 = -2+3i \qquad z_2 = 1+i$$

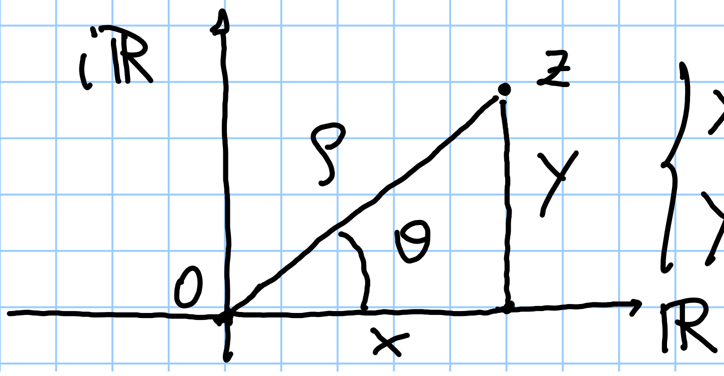
$$z_1 \cdot z_2 = -5+i$$

$z_1 \cdot z_2 = -5+i$



IN COORDINATE (X,Y) IL SIGNIFICATO GEOMETRICO DEL PRODOTTO NON E' CHIARO

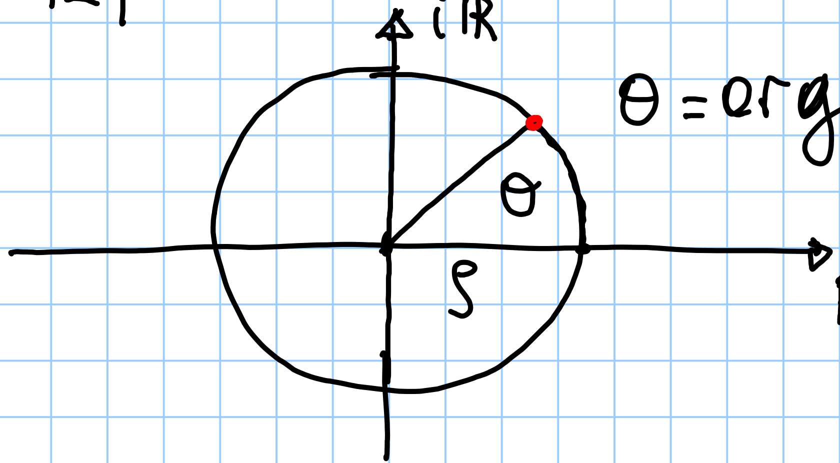
RAPPRESENTAZIONE TRIGONOMETRICA



$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$$

$0 = 0 + i0$

$\rho = |z| =$ MODULO di z



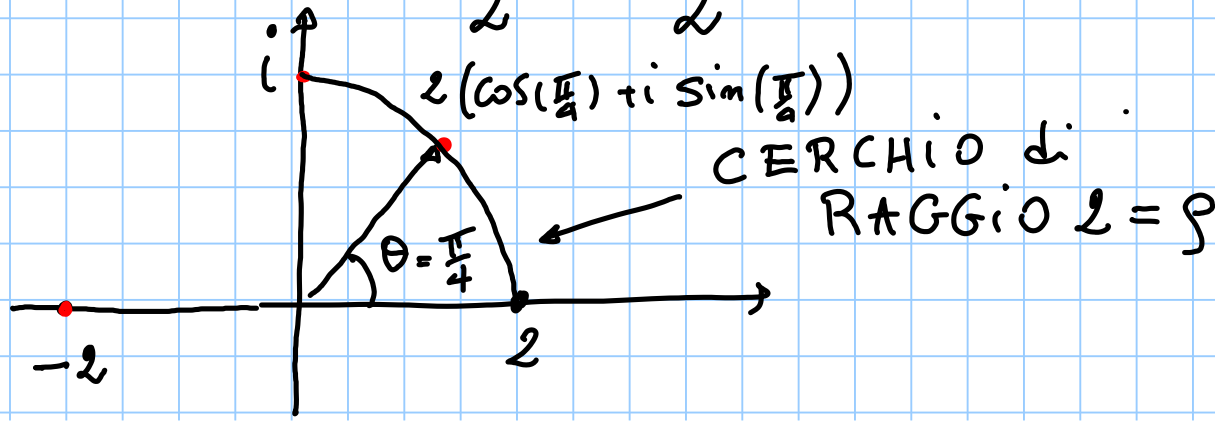
$\theta = \arg(z) =$ ARGOMENTO di z

$\rho \in [0, +\infty) \quad , \quad \theta \in [0, 2\pi)$

$$(\rho, \theta) \longrightarrow z = \rho(\cos\theta + i \sin\theta)$$
$$= \rho\cos\theta + i\rho\sin\theta$$

$\rho = 2 \quad , \quad \theta = \frac{\pi}{4}$

$$z = 2\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \sqrt{2} + i\sqrt{2}$$



$$\rho = 1, \theta = \frac{\pi}{2} \qquad z = 1 \cdot \left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)\right) = i$$

$$\rho = 2, \theta = \pi \qquad z = 2 \cdot \left(\cos(\pi) + i \sin(\pi)\right) = -2$$

$$\rho = 2, \theta = 0 \qquad z = 2\left(\cos(0) + i \sin(0)\right) = 2$$

$$[0, +\infty) = \{ z \in \mathbb{C} : \theta = 0 \}$$

$$(-\infty, 0] = \{ z \in \mathbb{C} : \theta = \pi \}$$

$$i \cdot [0, +\infty) = \left\{ z \in \mathbb{C} : \theta = \frac{\pi}{2} \right\}$$

$$i \cdot (-\infty, 0] = \left\{ z \in \mathbb{C} : \theta = \frac{3}{2}\pi \right\}$$

FORMULA di EULERO

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$$\theta \in \mathbb{R}$$

$$z = \rho(\cos(\theta) + i\sin(\theta)) = \rho e^{i\theta}$$

$$z = e^{i\frac{\pi}{2}} = i \quad ; \quad z = e^{i\cdot 0} = 1 ;$$

$$z = 2e^{i\frac{\pi}{4}} = 2\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \sqrt{2} + i\sqrt{2} ;$$

$$z = 2e^{i\pi} = 2(-1 + i0) = -2 \quad ;$$

D'ALTRA PARTE SI HA

$$e^{2\pi i} = \cos(2\pi) + i\sin(2\pi) = 1$$

$$e^{-4\pi i} = \cos(-4\pi) + i\sin(-4\pi) = 1$$

$$e^{2k\pi i} = \cos(2k\pi) + i\sin(2k\pi) = 1$$

$$\forall k \in \mathbb{Z}$$

LA RAPPRESENTAZIONE TRIGONOMETRICA

NON E' UNICA

$$z = e^{i\frac{\pi}{2}} = i \quad , \quad z = e^{i\left(\frac{\pi}{2}+2\pi\right)} =$$

$$= e^{i\frac{5}{2}\pi} = i$$

$$z = e^{i\left(\frac{\pi}{2}-6\pi\right)} =$$

$$= e^{-i\frac{11}{2}\pi} = i$$

$$z = \rho\left(\cos(\theta) + i\sin(\theta)\right) =$$

$$= \rho\left(\cos(\theta+2k\pi) + i\sin(\theta+2k\pi)\right)$$

$$= \rho e^{i\theta} = \rho e^{i(\theta+2k\pi)} \quad , \quad k \in \mathbb{Z}$$