$$R(x,y) = \frac{P(x,y)}{Q(x,y)}$$

P e Q polinomi in x a coefficienti polinomi in y

$$P(x,y) = 1 + xy + y^4$$

$$Q(x,y) = 3 + x + x^2 y^2 + x^3$$

$$\int R\left(x, \sqrt[m]{\frac{ax+b}{cx+d}}\right) dx$$

SOSTITUZIONE

RAZIONALIZZANTE

$$t = \sqrt[m]{\frac{ax+b}{cx+d}}$$

$$\int \frac{dx}{x\sqrt{2-x}} =$$

$$t = \sqrt{2-x}$$

$$2-x > 0$$

$$t^2 = 2-x \quad , \quad x = 2-t^2$$

$$dx = -2t\,dt$$

$$\int \frac{-2t\,dt}{(2-t^2)\,t} =$$

$$= 2\int \frac{dt}{t^2-2} = \ldots\ldots$$

$$= \frac{\sqrt{2}}{2} \log \frac{|t-\sqrt{2}|}{|t+\sqrt{2}|} + C =$$

$$= \frac{\sqrt{2}}{2}\log\frac{\left|\sqrt{2-x}-\sqrt{2}\right|}{\left|\sqrt{2+x}+\sqrt{2}\right|}+c$$

$$\int_0^2 \frac{4}{x+2}\sqrt[3]{\frac{2-x}{x+2}}\,dx$$

$t=g(x)$

$x\in[0,2]$

g invertibile

$$t=\sqrt[3]{\frac{2-x}{x+2}}\ ;\quad t^3=\frac{2-x}{x+2}$$

$$2-x=t^3x+2t^3$$

$$x(t^3+1)=2-2t^3$$

$$x=2\,\frac{1-t^3}{1+t^3}\ ;$$

$$dx=2\,\frac{-3t^2(1+t^3)-3t^2(1-t^3)}{(1+t^3)^2}\,dt=$$

$$= 2\,\frac{-6t^2\,dt}{(1+t^3)^2} = -12\,\frac{t^2}{(1+t^3)^2}dt$$

$$\frac{4}{x+2} = \frac{4}{2\cdot\frac{1-t^3}{1+t^3}+2} = \frac{4(1+t^3)}{2\left[1-t^3+1+t^3\right]}$$

$$= 1+t^3 .$$

$$\int_1^0 (1+t^3)\cdot t\,(-12)\,\frac{t^2}{(1+t^3)^2}\,dt =$$

$$t = \sqrt[3]{\frac{2-x}{x+2}} \qquad \begin{array}{l} x=0 \longrightarrow t=1 \\ x=2 \longrightarrow t=0 \end{array}$$

$$12\int_0^1 \frac{t^3}{1+t^3}dt = 12\int_0^1 \frac{1+t^3-1}{1+t^3}dt$$

$$= 12\int_0^1 dt - 12\int_0^1 \frac{1}{1+t^3}\,dt$$

$$= 12 - 12\int_0^1 \frac{dt}{1+t^3}$$

$(1+t^3)$; $t_1 = -1$

$(1+t^3) = (t - t_1)(at^2 + bt + c)$

$$\frac{1-x^3}{1-x} = 1 + x + x^2$$

$$\frac{1+t^3}{1+t} = \frac{1-(-t)^3}{1-(-t)} = 1 - t + t^2$$

$$1+t^3 = (t+1)(t^2 - t + 1)$$

$\Delta < 0$

$$\frac{1}{t^3+1} = \frac{A_1}{t+1} + \frac{A_2 t + A_3}{t^2-t+1} \quad \dots$$

$$A_1 = \frac{1}{3}, \quad A_2 \ \dots, \quad A_3 \qquad \text{Ex}$$

$$\int \frac{1}{1+t^3}\,dt = \frac{1}{3}\log(t+1)$$

$$+ \frac{1}{\sqrt{3}} \operatorname{arctg}\left(\frac{2t-1}{\sqrt{3}}\right)$$

$$- \frac{1}{6} \log(1-t+t^2)$$

$$\int_0^2 \frac{4}{x+2} \sqrt[3]{\frac{2-x}{x+2}}\,dx =$$

$$12 \int_0^1 dt - 12 \int_0^1 \frac{1}{1+t^3}\,dt =$$

$$= 12 - 12 \left[\frac{1}{3}\log(2) + \frac{1}{\sqrt{3}} \operatorname{arctg}\left(\frac{1}{\sqrt{3}}\right) \right.$$

$$-\frac{1}{\sqrt{3}}\operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right)\Big] =$$

$$= 12 - 4\log(2) - 12\left[\frac{2}{\sqrt{3}}\operatorname{arctg}\left(\frac{1}{\sqrt{3}}\right)\right]$$

$$= 12 - 4\log(2) - \frac{24}{\sqrt{3}}\,\frac{\pi}{6}.$$

$$\operatorname{arctg}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$$

$$\sin(\pi/6) = \frac{1}{2}$$

$$\cos(\pi/6) = \frac{\sqrt{3}}{2} \qquad \operatorname{Tg}(\pi/6) = \frac{1}{\sqrt{3}}$$

$$\int_1^{16} \frac{dx}{x^{5/4}\left(1+\sqrt[4]{x}\right)} = \qquad t = \sqrt[4]{x},\ x \geq 0$$

$$x = t^4$$

$$t = g(x),\ x \in [1,16] \quad dx = 4t^3\,dt$$

g invertibile

$$x = 1 \qquad t = 1$$

$$x = 16 \qquad t = 2$$

$$= \int_1^2 \frac{4t^3\,dt}{t^5(1+t)} = \int_1^2 \frac{4\,dt}{t^2(1+t)} =$$

$t_1 = t_2 = 0,\ t_3 = -1$

$$= 4\int_1^2 \frac{1+t-t}{t^2(1+t)}\,dt =$$

$$= 4\int_1^2 \frac{1}{t^2}\,dt - 4\int_1^2 \frac{dt}{t(1+t)} = \dots$$

Ex.

$$= 4\left(\frac{1}{2} + \log\left(\frac{3}{4}\right)\right) = 2 + 4\log\left(\frac{3}{4}\right)$$

$$\int R(\sin(x), \cos(x))\,dx \quad \Bigg| \begin{array}{l} t = g(x) \\ g \text{ invertibile} \end{array}$$

$$t = \mathrm{Tg}\left(\frac{x}{2}\right), \quad -\pi < x < \pi$$

in

$$x = 2\arctg(t), \quad dx = \frac{2\,dt}{1+t^2}$$

$$\sin(x) = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) =$$

$$= 2\,\mathrm{Tg}\left(\frac{x}{2}\right)\frac{\cos^2\left(\frac{x}{2}\right)}{\sin^2\left(\frac{x}{2}\right)+\cos^2\left(\frac{x}{2}\right)}$$

$$= 2\frac{t}{1+t^2}$$

$$\cos(x) = 1 - 2\frac{\sin^2\left(\frac{x}{2}\right)}{\sin^2\left(\frac{x}{2}\right)+\cos^2\left(\frac{x}{2}\right)}$$

$$= 1 - 2\frac{t^2}{t^2+1} = \frac{1-t^2}{1+t^2}$$

$$\int \frac{dx}{1+\sin(x)+2\cos(x)} =$$

$$= \int \frac{1}{1 + \frac{2t}{1+t^2} + 2\frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\, dt =$$

$$= 2\int \frac{dt}{1+t^2+2t+2-2t^2} =$$

$$= 2\int \frac{dt}{3+2t-t^2} = \cdots\cdots =$$

$$= \frac{1}{2} \log \frac{\left| \operatorname{tg}\left(\frac{x}{2}\right)+1 \right|}{\left| \operatorname{tg}\left(\frac{x}{2}\right)-3 \right|} + c$$

$$\int \frac{dx}{\cos(x)} = \int \frac{\cos(x)}{\cos^2(x)}\,dx =$$

$$= \int \frac{\cos(x)\,dx}{1-\sin^2(x)} = \qquad t = \sin(x)$$

$$= \int \frac{dt}{1-t^2} = \dots$$

$$\dots = \frac{1}{2}\log\frac{|1+\sin(x)|}{|1-\sin(x)|} + c$$

$$x \longrightarrow x$$

$$\int \frac{\sin(x)\cos(x)}{\sqrt{3+\cos(2x)}}\,dx$$

$$3+\cos(2x) = t \qquad \dots$$

Ex

$$\int R\left(\cos^2(x), \sin(x)\cos(x), \sin^2(x)\right)dx$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$$

$$\int R\left(\cos^2(x), \sin(x)\cos(x), \sin^2(x)\right)dx =$$

$$= \int \tilde{R}\left(\cos(2x), \sin(2x) \right) dx$$

$$= \int \overline{R}\left(\cos(y), \sin(y) \right) dy$$

$$t = \operatorname{tg}\left(\frac{y}{2}\right)$$

ALTRO METODO

$$t = tg(x) \quad ;$$

$$x = arcTg(t) \longrightarrow dx = \frac{dt}{1+t^2}$$

$$\sin^2(x) = \frac{\sin^2(x)}{\sin^2(x)+\cos^2(x)} =$$

$$= \frac{Tg^2(x)}{Tg^2(x)+1} = \frac{t^2}{1+t^2}$$

$$\cos^2(x) = \frac{1}{1+t^2}$$

$$\sin(x)\cos(x) = \frac{\sin(x)\cos(x)}{\sin^2(x)+\cos^2(x)} =$$

$$= \frac{t}{1+t^2}$$

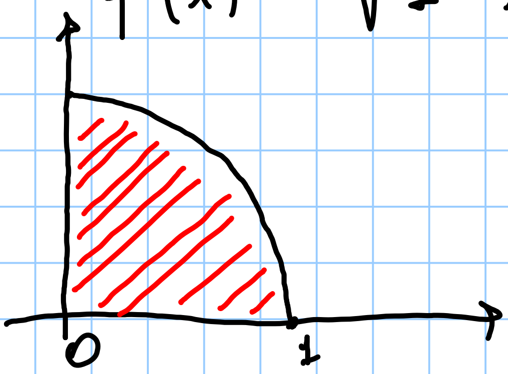
$$\int \frac{dx}{\sin^2(x) + 2\cos^2(x)} = \ldots =$$

Ex

$$= \frac{1}{\sqrt{2}} \operatorname{arctg}\left(\frac{\operatorname{tg}(x)}{\sqrt{2}}\right) + C$$

L'AREA DEL CERCHIO

$$f(x) = \sqrt{1-x^2}$$



$$A = \{(x,y) \in \mathbb{R}^2 : x \geq 0, y \geq 0, x^2+y^2 \leq 1\}$$

$$A = \left\{ (x,y) \in \mathbb{R}^2 : 0 \leq y \leq \sqrt{1-x^2},\ x \in [0,1] \right\}$$

$$|A| = \int_0^1 \sqrt{1-x^2}\,dx \quad ; \quad t \in [0, \tfrac{\pi}{2}]$$

$$x = \sin(t)$$

$$x = 0 \quad t = 0$$

$$x = 1 \quad t = \tfrac{\pi}{2}$$

$$dx = \cos(t)\,dt$$

$$\sqrt{1-\sin^2(t)} = \sqrt{\cos^2(t)} = |\cos(t)|$$

$$t \in [0, \tfrac{\pi}{2}] \qquad = \cos(t)$$

$$|A| = \int_0^{\frac{\pi}{2}} \cos(t)\cos(t)\,dt = \int_0^{\frac{\pi}{2}} \cos^2(t)\,dt$$

$$\int_0^{\frac{\pi}{2}} \cos^2(t)\,dt = \int_0^{\pi/2} \cos(t)\,(\sin(t))'\,dt$$

$$= \cos(t)\sin(t)\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin(t)\left(-\sin(t)\right)dt$$

$$= \int_0^{\frac{\pi}{2}} \sin^2(t)\,dt = \int_0^{\frac{\pi}{2}} \left(1-\cos^2(t)\right)dt$$

$$\boxed{I} = \int_0^{\frac{\pi}{2}} \cos^2(t)\,dt = \int_0^{\frac{\pi}{2}} dt - \int_0^{\frac{\pi}{2}} \cos^2(t)\,dt$$

$$= \frac{\pi}{2} - \boxed{I} \quad ; \quad I = \frac{\pi}{2} - I$$

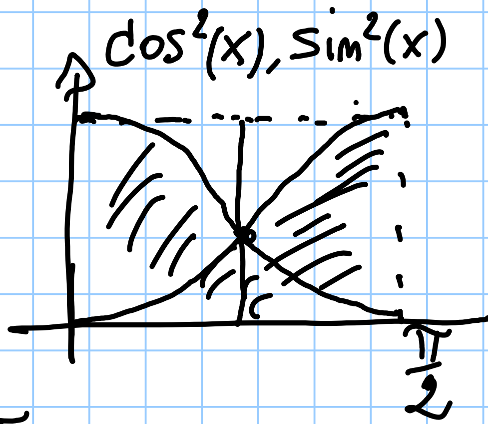
$$2I = \frac{\pi}{2} \Longrightarrow I = \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{2}} \cos^2(t)\,dt =$$

$$\cos^2(t) = \frac{1+\cos(2t)}{2}$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2}\cos(2t)\right)dt =$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} dt = \frac{\pi}{4}$$



$$2\int_0^{\frac{\pi}{2}} \cos^2(x)\,dx =$$

$$\int_0^{\frac{\pi}{2}} \cos^2(x)\,dx + \int_0^{\frac{\pi}{2}} \sin^2(x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\cos^2(x) + \sin^2(x)\right) dx$$

$$= \int_0^{\frac{\pi}{2}} dx = \pi/2;$$

$$\int_0^{\pi/2} \cos^2(x)\,dx = \frac{\pi}{4}$$

$$I(x) = \int \sqrt{1-x^2}\,dx = \int \sqrt{1-x^2}\,(x)'\,dx =$$

$$x\sqrt{1-x^2} - \int x\,\frac{(-2x)}{2\sqrt{1-x^2}}\,dx =$$

$$= x\sqrt{1-x^2} + \int \frac{x^2}{\sqrt{1-x^2}}\,dx =$$

$$= x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}}\,dx$$

$$= x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}\,dx =$$

$$= x\sqrt{1-x^2} - \int \sqrt{1-x^2} + \int \frac{dx}{\sqrt{1-x^2}}$$

$$= x\sqrt{1-x^2} - I(x) + \arcsin(x)$$

$$2I(x) = x\sqrt{1-x^2} + \arcsin(x) + c$$

$$\int \sqrt{1-x^2} = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin(x) + c$$

$$\int \sqrt{1-x^2}\,dx =$$

$$x = \sin(t)$$

$$-\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

$$\int \sqrt{1-\sin^2(t)}\,\cos(t)\,dt =$$

$$\sqrt{1-\sin^2(t)} = \sqrt{\cos^2(t)} = |\cos(t)|$$

$$\left\{ t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right\} \qquad = \cos(t)$$

$$I(t) = \boxed{\int \cos^2(t)\,dt} = \int \cos(t)(\sin(t))'\,dt$$

$$= \sin(t)\cos(t) + \int \sin^2(t)\,dt$$

$$= \sin(t)\cos(t) + \int \left(1-\cos^2(t)\right)dt$$

$$= \sin(t)\cos(t) + t - \underbrace{\int \cos^2(t)\,dt}_{I}$$

$$I = t + \sin(t)\cos(t) - I$$

$$2I = t + \sin(t)\cos(t)$$

$$\int \cos^2(t)\,dt = \frac{t}{2} + \frac{1}{2}\sin(t)\cos(t)$$

$$+c = \frac{t}{2} + \frac{1}{4}\sin(2t) + c;$$

$$\int \sqrt{1-x^2} = \int \cos^2(t) =$$

$$x = \sin(t), \quad t = \arcsin(x)$$

$$-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

$$= \frac{1}{2}\arcsin(x) + \frac{1}{2}x\sqrt{1-x^2} + c$$

$$\sin(t) = x \qquad \cos(t) = \sqrt{1-x^2}$$

$$I = \int_0^{\frac{\pi}{2}} \sin^4(x)\,dx =$$

$$\boxed{\int (\sin(x))^{2m}(\cos(x))^{2m}\,dx} = \int_0^{\frac{\pi}{2}} \sin^3(x)\,(-\cos(x))'\,dx$$

$$= -\sin^3(x)\cos(x)\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos^2(x)\cdot 3\sin^2(x)$$

$$= 3\int_0^{\frac{\pi}{2}} \cos^2(x)\sin^2(x)\,dx =$$

$$= 3\int_0^{\frac{\pi}{2}} (1-\sin^2(x))\sin^2(x)\,dx =$$

$$= 3\int_0^{\frac{\pi}{2}} \sin^2(x)\,dx - 3\int_0^{\frac{\pi}{2}} \sin^4(x)\,dx$$

$$= 3\int_0^{\frac{\pi}{2}} \sin^2(x)\,dx - 3I$$

$$I = 3\int_0^{\frac{\pi}{2}} \sin^2(x)\,dx - 3I$$

$$4I = 3\int_0^{\frac{\pi}{2}} \sin^2(x)\,dx$$

$$I = \frac{3}{4}\cdot\int_0^{\frac{\pi}{2}} \sin^2(x)\,dx$$

$$= \frac{3}{16}\pi$$

$$\int_0^{\frac{\pi}{2}} \sin^4(x)\,dx = \begin{cases} \sin^2(x) = \dfrac{1-\cos(2x)}{2} \\ \underline{\text{Ex}} \end{cases}$$

$$\int_0^{\frac{\pi}{2}} \cos^2(x)\sin^3(x)\,dx = \boxed{\int \cos^{2m}(x)\,\sin^{2m+1}(x)\,dx}$$

$$= \int_0^{\frac{\pi}{2}} \cos^2(x)\sin^2(x)\left(-\cos(x)\right)'\,dx$$

$$= -\int_0^{\frac{\pi}{2}} \cos^2(x)\left(1-\cos^2(x)\right)\left(\cos(x)\right)' dx$$

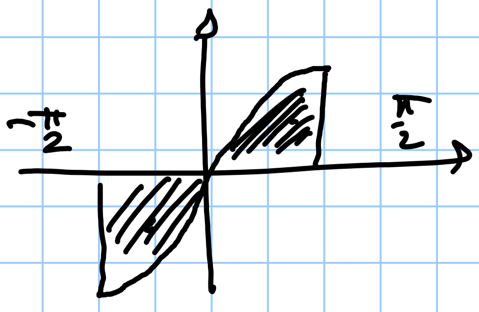
$\cos(x) = t$

$$= -\int_1^0 t^2(1-t^2)\, dt =$$

$$= \int_0^1 (t^2-t^4)\,dt = \frac{t^3}{3}-\frac{t^5}{5}\Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(x)\sin^3(x)\,dx = 0$$



$f : [-a, a] \to \mathbb{R}$,

INTEGRABILE

DISPARI

ALLORA $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$

DIM.

$$I = \int_{-a}^{a} f(x)\,dx \underset{t=-x}{=} \int_{a}^{-a} f(-t)\cdot(-dt) =$$

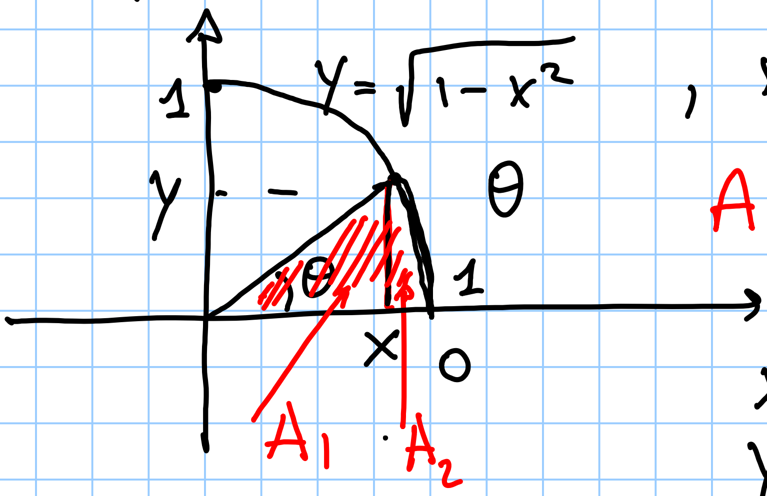
$$= \int_{a}^{-a} f(t)\,dt =$$

$$= -\int_{-a}^{a} f(t)\,dt = -I$$

$$2I = 0 \qquad \boxed{I = 0}$$

FACOLTATIVO $\to$ SIGNIFICATO GEOMETRICO ?

$$(\cos(\theta), \sin(\theta)) = (x, y)$$



$$, \quad x^2 + y^2 = 1$$

$$A = A_1 + A_2 = \frac{\theta}{2}$$

$$x = \cos(\theta)$$
$$y = \sin(\theta)$$

$$A_1 = \frac{x \cdot y}{2} = \frac{x\sqrt{1-x^2}}{2} = \frac{1}{2}\cos(\theta)\sin(\theta)$$

$$A_2 = \int_x^1 \sqrt{1-s^2}\, ds =$$

$$\left[\frac{1}{2}\arcsin(s) + \frac{1}{2} s\sqrt{1-s^2}\right]_x^1$$

$$= \frac{1}{2}\underbrace{\arcsin(1)}_{= \frac{\pi}{2}} - \frac{1}{2}\arcsin(x) - \frac{1}{2} x\sqrt{1-x^2} =$$

$$\frac{\pi}{4} - \frac{1}{2}\arcsin(\cos(\theta)) - \frac{1}{2}\cos(\theta)\sqrt{1-\cos^2(\theta)}$$

$$= \frac{\pi}{4} - \frac{1}{2}\arcsin\left(\sin\left(\frac{\pi}{2}-\theta\right)\right) - \frac{1}{2}\cos(\theta)\sin(\theta)$$

$$= \frac{\pi}{4} - \frac{1}{2}\left(\frac{\pi}{2}-\theta\right) - \frac{1}{2}\cos(\theta)\sin(\theta)$$

$$A_1 + A_2 = \cancel{\frac{1}{2}\cos(\theta)\sin(\theta)} +$$

$$\cancel{\frac{\pi}{4}} - \cancel{\frac{\pi}{4}} + \frac{\theta}{2} - \cancel{\frac{1}{2}\cos(\theta)\sin(\theta)} = \frac{\theta}{2}$$

$$A = \theta/2$$