$$\int \frac{kx+h}{ax^2+bx+c}\,dx \quad , \qquad a \neq 0$$

$$\Delta = b^2 - 4ac \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases}$$

$\boxed{\Delta > 0}$ $\qquad \exists \{x_1, x_2\} \subset \mathbb{R}, \ (x_1 \neq x_2)$:

$$ax^2+bx+c = a(x-x_1)(x-x_2).$$

$\exists$ e sono unici A_1 e A_2 :

$$\frac{kx+h}{ax^2+bx+c} = \frac{A_1}{x-x_1} + \frac{A_2}{x-x_2}$$

$$\int \frac{kx+h}{ax^2+bx+c}\,dx = \int \left(\frac{A_1}{x-x_1} + \frac{A_2}{x-x_2} \right) dx$$

$$= A_1 \log|x-x_1| + A_2 \log|x-x_2| + c \quad \blacksquare$$

$$\int \frac{dx}{3x^2+6x-24} \;;$$

$$x_1 = 2 \;, \quad x_2 = -4$$

$$\frac{1}{3x^2+6x-24} = \frac{1}{3(x-2)(x+4)} =$$

$$\frac{0 \cdot x + 1/3}{(x-2)(x+4)} = \frac{A_1}{x-2} + \frac{A_2}{x+4} =$$

$$= \frac{A_1(x+4) + A_2(x-2)}{(x-2)(x+4)} =$$

$$= \frac{(A_1 + A_2)x + 4A_1 - 2A_2}{(x-2)(x+4)}$$ ← Polinomio GRADO 1

$$\begin{cases} A_1 + A_2 = 0 \\ 4A_1 - 2A_2 = \frac{1}{3} \end{cases} \Rightarrow \begin{cases} A_2 = -A_1 \\ 4A_1 + 2A_1 = \frac{1}{3} \end{cases}$$

$$\begin{cases} A_2 = -\frac{1}{18} \\ A_1 = \frac{1}{18} \end{cases}$$

$$\int \frac{dx}{3x^2 + 6x - 24} = \int \left(\frac{1}{18} \frac{1}{x-2} - \frac{1}{18} \frac{1}{x+4} \right) dx$$

$$= \frac{1}{18} \log|x-2| - \frac{1}{18} \log|x+4| + c$$

$$= \frac{1}{18} \log \frac{|x-2|}{|x+4|} + c$$

$$\boxed{\Delta = 0} \qquad \exists\, x_1 \in \mathbb{R},$$

$$ax^2+bx+c = a(x-x_1)^2$$

$\exists$ e sono unici A_1 e A_2

$$\frac{kx+h}{ax^2+bx+c} = \frac{A_1}{(x-x_1)} + \frac{A_2}{(x-x_1)^2}$$

$$\int \frac{kx+h}{ax^2+bx+c}\,dx = \int \left(\frac{A_1}{x-x_1} + \frac{A_2}{(x-x_1)^2} \right) dx$$

$$= A_1 \log|x-x_1| - \frac{A_2}{x-x_1} + c$$

$$\int \frac{3x \quad dx}{2x^2-4x+2} = \int \frac{3x \quad dx}{2(x-1)^2}$$

$$\{x_1 = x_2 = 1\}$$

$$= \frac{3}{2} \int \frac{x}{(x-1)^2} \, dx$$

$$\frac{x}{(x-1)^2} = \frac{A_1}{(x-1)} + \frac{A_2}{(x-1)^2} =$$

$$= \frac{A_1(x-1) + A_2}{(x-1)^2} = \frac{A_1 x + A_2 - A_1}{(x-1)^2}$$

$$\begin{cases} A_1 = 1 \\ A_2 - A_1 = 0 \end{cases} \qquad \begin{array}{l} A_1 = 1 \\ A_2 = A_1 = 1 \end{array}$$

$$\int \frac{3x}{2x^2 - 4x + 2} \, dx = \frac{3}{2} \int \frac{x}{(x-1)^2} \, dx =$$

$$= \frac{3}{2} \int \left(\frac{1}{x-1} + \frac{1}{(x-1)^2} \right) dx$$

$$= \frac{3}{2} \log|x-1| - \frac{3}{2} \frac{1}{(x-1)} + c$$

$$\int \frac{3x}{2x^2-4x+2}\,dx = \frac{3}{2}\int \frac{x}{(x-1)^2}\,dx$$

$$= \frac{3}{2}\int \frac{(x-1)+1}{(x-1)^2}\,dx =$$

$$\frac{3}{2}\int \left(\frac{1}{x-1} + \frac{1}{(x-1)^2}\right)dx = \ldots$$

$$\boxed{\Delta < 0} \qquad \exists\, x_1 \in \mathbb{R},\ \exists\, y_1 \in \mathbb{R}\setminus\{0\}$$

$$ax^2+bx+c = a\left[(x-x_1)^2+y_1^2\right]$$

$$\int \frac{dx}{x^2+\alpha^2} = \frac{1}{\alpha^2}\int \frac{dx}{1+\left(\frac{x}{\alpha}\right)^2} =$$

$\alpha \neq 0$

$$t = \frac{x}{\alpha} \qquad dx = \alpha\, dt$$

$$= \frac{1}{\alpha^{\cancel{2}}} \int \frac{\cancel{\alpha}\, dt}{1+t^2} = \frac{1}{\alpha} \operatorname{arctg}(t) + c$$

$$= \frac{1}{\alpha} \operatorname{arctg}\left(\frac{x}{\alpha}\right) + c \quad \blacksquare$$

$$\int \frac{x}{x^2+\alpha^2}\, dx \underset{x^2+\alpha^2 = t}{=} \ldots\ldots =$$

$$\frac{1}{2} \int \frac{2x}{x^2+\alpha^2}\, dx = \frac{1}{2} \int \frac{(x^2+\alpha^2)'}{x^2+\alpha^2}\, dx$$

$$\frac{1}{2} \log\left(x^2+\alpha^2\right) + c$$

$$\boxed{\Delta < 0} \qquad ax^2+bx+c = a\left[(x-x_1)^2+y_1^2\right]$$

$$\int \frac{dx}{ax^2+bx+c} = \int \frac{dx}{a\left[(x-x_1)^2+y_1^2\right]} =$$

$$= \frac{1}{a}\int \frac{dx}{(x-x_1)^2+y_1^2} =$$

$$x-x_1 = t \qquad dx = dt$$

$$= \frac{1}{a}\int \frac{dt}{t^2+y_1^2} = \left[\ \dots\ \right] =$$

$$= \frac{1}{a}\,\frac{1}{y_1}\,\operatorname{arctg}\left(\frac{t}{y_1}\right)+c = \frac{1}{ay_1}\operatorname{arctg}\left(\frac{x-x_1}{y_1}\right)+c$$

$$\int \frac{dx}{x^2-4x+20}$$

$$\cancel{x^2}-4x = (x-x_1)^2-x_1^2$$

$$= \cancel{x^2} - 2xx_1 + \cancel{x_1^2} - \cancel{x_1^2}$$

$$-4 = -2x_1 \qquad \boxed{x_1 = 2}$$

$$x^2 - 4x + 20 = (x-2)^2 - 4 + 20$$

$$= (x-2)^2 + 16$$

$$\Longrightarrow y_1^2 = 16 \; , \; y_1 = \pm 4$$

$$\int \frac{dx}{x^2 - 4x + 20} = \int \frac{dx}{(x-2)^2 + 16} =$$

$$= \frac{1}{4} \operatorname{arctg}\left(\frac{x-2}{4}\right) + c$$

METODO EQUIVALENTE:

$$\cancel{x^2} - 4x + 20 = [(x - x_1)^2 + y_1^2]$$

$$= \cancel{x^2} - 2xx_1 + x_1^2 + y_1^2$$

$$-4 = -2x_1 \longrightarrow x_1 = 2$$

$$20 = 4 + y_1^2 \longrightarrow y_1^2 = 16 \; , \; y_1 = \pm 4$$

NOTA: SI PUÒ DIMOSTRARE CHE SE ax^2+bx+c; $\Delta = b^2-4ac<0$

ALLORA $$ax^2+bx+c = a\left[(x-x_1)^2+y_1^2\right].$$

$$x_1 = -\frac{b}{2a}, \quad y_1^2 = \frac{|\Delta|}{4a^2}.$$

In questo caso si hanno RADICI COMPLESSE CONIUGATE $z_\pm = x_1 \pm i y_1$

NOTA: se $\Delta > 0$ si ottiene

$$ax^2+bx+c = a\left[(x-x_1)^2 - y_1^2\right], \quad y_1 \neq 0$$

$$\int \frac{dx}{2x^2 - 2\sqrt{2}\,x + 3}$$

$$\cancel{2x^2} - 2\sqrt{2}\,x + 3 = 2\left[(x-x_1)^2 + y_1^2\right]$$

$$= \cancel{2x^2} - 4xx_1 + 2x_1^2 + 2y_1^2$$

$$-2\sqrt{2} = -4x_1 \qquad x_1 = \frac{\sqrt{2}}{2}$$

$$3 = 2\left(\frac{\sqrt{2}}{2}\right)^2 + 2y_1^2 \; ; \; y_1^2 = 1$$

$$2x^2 - 2\sqrt{2}\,x + 3 = 2\left[\left(x - \frac{\sqrt{2}}{2}\right)^2 + 1\right]$$

$$\int \frac{dx}{2x^2 - 2\sqrt{2}\,x + 3} =$$

$$\int \frac{dx}{2\left[\left(x - \frac{\sqrt{2}}{2}\right)^2 + 1\right]} = \frac{1}{2}\operatorname{arctg}\left(x - \frac{\sqrt{2}}{2}\right) + C$$

$$k\int \frac{2x}{ax^2+bx+c}\,dx =$$

$$\frac{k}{a}\int \frac{D(ax^2)}{ax^2+bx+c}\,dx =$$

$$\frac{k}{a}\int \frac{D(ax^2)\quad +b \quad -b}{ax^2+bx+c}\,dx$$

$$\frac{k}{a}\int \frac{D(ax^2+bx+c) \quad -b}{ax^2+bx+c}\,dx$$

$$= \frac{k}{a}\int \frac{(ax^2+bx+c)'}{ax^2+bx+c}\,dx$$

$$- \frac{kb}{a}\int \frac{1}{ax^2+bx+c}\,dx$$

$$= \frac{k}{a} \log | a x^2 + b x + c | -$$

$$- \frac{k b}{a} \int \frac{1}{a x^2 + b x + c} \, dx$$

$$3 \int \frac{x}{x^2 - 4x + 20} \, dx =$$

$$= \frac{3}{2} \int \frac{(x^2 - 4x)' + 4}{x^2 - 4x + 20} \, dx$$

$$= \frac{3}{2} \int \frac{(x^2 - 4x + 20)'}{x^2 - 4x + 20} dx + 6 \int \frac{dx}{x^2 - 4x + 20}$$

$$= \frac{3}{2} \log (x^2 - 4x + 20) + \frac{6}{4} \operatorname{arctg} \left(\frac{x - 2}{4} \right)$$

$$+ C$$

MODI EQUIVALENTI

$$\int \frac{x}{ax^2+bx+c}\,dx =$$

$$ax^2+bx+c = t$$

$$(2ax+b)\,dx = dt$$

$$\frac{1}{2a}\int \frac{2ax+b-b}{ax^2+bx+c}\,dx =$$

$$= \frac{1}{2a}\int \frac{(2ax+b)\,dx}{ax^2+bx+c} - \frac{b}{2a}\int \frac{dx}{ax^2+bx+c} =$$

$$= \frac{1}{2a}\log|ax^2+bx+c| - \frac{b}{2a}\int \frac{dx}{ax^2+bx+c}$$

$$\int \frac{x}{ax^2+bx+c}\,dx = \frac{1}{a}\int \frac{x}{(x-x_1)^2+y_1^2}\,dx$$

$$= \qquad = \frac{1}{a}\int \frac{t+x_1}{t^2+y_1^2}\,dt =$$

$x - x_1 = t$

$dx = dt$

$$= \frac{1}{a}\int \frac{t}{t^2+y_1^2}\,dt + \frac{1}{a}\int \frac{x_1}{t^2+y_1^2}\,dt$$

$$= \frac{1}{2a}\log\left(t^2+y_1^2\right) + \frac{1}{a}\int \frac{x_1}{t^2+y_1^2}\,dt$$

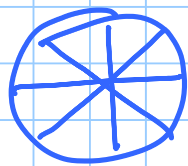
$$= \frac{1}{2a}\log\left((x-x_1)^2+y_1^2\right) + \frac{x_1}{ay_1}\operatorname{arcTg}\left(\frac{x-x_1}{y_1}\right) + C$$

$$= \frac{1}{2a}\log\left(\frac{ax^2+bx+c}{a}\right) + \frac{x_1}{ay_1}\operatorname{arctg}\left(\frac{x-x_1}{y_1}\right) + C$$

$$= \frac{1}{2a} \log |ax^2 + bx + c| + \frac{x_1}{a y_1} \operatorname{arctg}\left(\frac{x - x_1}{y_1}\right)$$

$$+ c$$

$ax^2 + bx + c \quad , \quad \Delta > 0$



$$ax^2 + bx + c = a\left[(x - x_1)^2 - y_1^2\right]$$

$$\int \frac{dx}{ax^2 + bx + c} = \frac{1}{a} \int \frac{dx}{(x - x_1)^2 - y_1^2} =$$

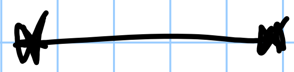
$x - x_1 = t$

$$= \frac{1}{a} \int \frac{dt}{t^2 - y_1^2} = \frac{1}{a y_1^2} \int \frac{dt}{\left(t/y_1\right)^2 - 1} =$$

$s = t/y_1$

$$= \frac{1}{a y_1} \int \frac{d\left(t/y_1\right)}{\left(t/y_1\right)^2 - 1} = \frac{1}{a y_1} \int \frac{ds}{s^2 - 1} =$$

$$\frac{1}{2ay_1}\log\frac{|s-1|}{|s+1|}+c=\frac{1}{2ay_1}\log\frac{|x-x_1-y_1|}{|x-x_1+y_1|}+c$$



$$\int\frac{x+2}{2x^2+4x+7}\,dx$$

Ex

CASO GENERALE

$$\int \frac{P(x)}{Q(x)}\,dx$$

$P(x)$ Polinomio GRADO n

$Q(x)$ Polinomio GRADO m,

$$n > m$$

$$\frac{P(x)}{Q(x)}, \quad \begin{matrix} \text{GRADO di } P \text{ è } n > \\ \text{GRADO di } Q \text{ è } m \end{matrix}$$

$$\frac{P(x)}{Q(x)} = q(x) + \frac{r(x)}{Q(x)},$$

GRADO di $q = n - m$

GRADO di $r < m$

$$\int \frac{x^5 + 2x^4 - 9x^3 - x^2 + 11x - 7}{x^2 + 2x - 8}\, dx$$

$$\begin{array}{rrrrrr|l}
x^5 & +2x^4 & -9x^3 & -x^2 & +11x & -7 & x^2+2x-8 \\ \cline{7-7}
x^5 & +2x^4 & -8x^3 & & & & x^3 - x + 1 \\ \cline{1-6}
0 & 0 & -x^3 & -x^2 & +11x & -7 & q(x) \\
 & & -x^3 & -2x^2 & +8x & & \text{GRADO} = 5-2=3 \\ \cline{3-6}
 & & 0 & x^2 & +3x & -7 &
\end{array}$$

x^2+2x-8

$0 \quad x+1$ GRADO < 2

$r(x) = x+1$ GRADO < 2

$$\frac{x^5+2x^4-9x^3-x^2+11x-7}{x^2+2x-8} = \underbrace{x^3-x+1}_{q(x)}$$

$$+ \frac{\overbrace{x+1}^{r(x)}}{\underbrace{x^2+2x-8}_{Q(x)}} = \dots$$

Ex

$$\int \frac{r(x)}{Q(x)}\,dx$$

GRADO di $r \leq 2$

GRADO di $Q = 3$

$$Q(x) = ax^3 + bx^2 + cx + d, \quad a \neq 0$$

T. degli ZERI $\Longrightarrow$ Q ha ALMENO una RADICE REALE

$$\exists\, x_1 \in \mathbb{R} : Q(x_1) = 0$$

1. Q ha 3 RADICI REALI E DISTINTE

$$Q(x) = a(x-x_1)(x-x_2)(x-x_3)$$

$$\frac{r(x)}{Q(x)} = \frac{A_1}{x-x_1} + \frac{A_2}{x-x_2} + \frac{A_3}{x-x_3}$$

2. Q ha 3 RADICI REALI, 2 COINCIDENTI

$$Q(x) = a(x-x_1)(x-x_2)^2$$

$$\frac{r(x)}{Q(x)} = \frac{A_1}{(x-x_1)} + \frac{A_2}{(x-x_2)} + \frac{A_3}{(x-x_2)^2}$$

3. Q ha 3 RADICI COINCIDENTI

$$\frac{r(x)}{Q(x)} = \frac{A_1}{(x-x_1)} + \frac{A_2}{(x-x_1)^2} + \frac{A_3}{(x-x_1)^3}$$

④ Q ha 1 RADICE REALE e

2 COMPLESSE e CONIUGATE

$$x = x_1 \quad , \quad "z_\pm = x_2 \pm i y_2"$$

In questo caso si ha

$$\frac{Q(x)}{(x-x_1)} = \tilde{a}x^2 + \tilde{b}x + \tilde{c}$$

dove $\Delta = \tilde{b}^2 - 4\tilde{a}\tilde{c} < 0$

Quindi

$$Q(x) = a(x - x_1)\left((x - x_2)^2 + y_2^2\right)$$

$$\frac{r(x)}{Q(x)} = \frac{A_1}{(x-x_1)} + \frac{A_2 x + A_3}{(x-x_2)^2 + y_2^2}$$

$$\int \frac{dx}{1+x^3} \; ; \qquad \frac{1-z^3}{1-z} = 1 + z + z^2$$

$$z = -x \qquad \frac{1+x^3}{1+x} = 1 - x + x^2$$

$$1 + x^3 = (1+x)(1-x+x^2)$$
$$= (x+1)(x^2-x+1)$$

$$\frac{1}{1+x^3} = \frac{A_1}{x+1} + \frac{A_2 x + A_3}{x^2-x+1} =$$

$$= \frac{A_1 x^2 - A_1 x + A_1 + A_2 x + A_3 + A_2 x^2 + A_3 x}{1+x^3}$$

$$A_1 + A_2 = 0 \,, \quad -A_1 + A_2 + A_3 = 0 \,,$$

$A_1 + A_3 = 1$;

$$\begin{cases} A_3 = 1 - A_1 \\ A_2 = -A_1 \\ -A_1 - A_1 + 1 - A_1 = 0 \end{cases}$$

$A_1 = \frac{1}{3}$, $A_2 = -\frac{1}{3}$, $A_3 = \frac{2}{3}$

$$\int \frac{dx}{1+x^3} = \frac{1}{3}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{-x+2}{x^2-x+1} =$$

$= \ldots\ldots$ Ex $=$

$$\frac{1}{3}\log|x+1| - \frac{1}{6}\log(x^2-x+1) +$$
$$+\frac{1}{\sqrt{3}}\operatorname{arcTg}\left(\frac{2x-1}{\sqrt{3}}\right) + c$$

$$\frac{r(x)}{Q(x)}, \quad \begin{array}{l} r(x) \text{ GRADO } \leq 3 \\ Q(x) \text{ GRADO } = 4 \end{array}$$

Q 4 RADICI REALI E DISTINTE

$x_1 \neq x_2 \neq x_3 \neq x_4$

$$\frac{r(x)}{Q(x)} = \frac{A_1}{(x-x_1)} + \frac{A_2}{(x-x_2)} + \frac{A_3}{(x-x_3)} + \frac{A_4}{(x-x_4)}$$

Q 2 RADICI REALI E DISTINTE

2 COINCIDENTI $\quad x_1 \neq x_2 \neq x_3 = x_4$

$$\frac{r(x)}{Q(x)} = \frac{A_1}{(x-x_1)} + \frac{A_2}{(x-x_2)} + \frac{A_3}{x-x_3} + \frac{A_4}{(x-x_3)^2}$$

- 4 RADICI REALI e COINCIDENTI

.....

- 2 RADICI REALI e DISTINTE e 2 COMPLESSE CONIUGATE

$$\frac{P(x)}{Q(x)} = \frac{A_1}{(x-x_1)} + \frac{A_2}{(x-x_2)} + \frac{A_3 x + A_4}{(x-x_3)^2 + y_3^2}$$

- 2 RADICI REALI e COINCIDENTI e 2 COMPLESSE CONIUGATE

.....

- 4 RADICI COMPLESSE E CONIUGATE DISTINTE

$$\frac{P(x)}{Q(x)} = \frac{A_1 x + A_2}{(x-x_1)^2 + y_1^2} + \frac{A_3 x + A_4}{(x-x_2)^2 + y_2^2}$$

• 2 RADICI COMPLESSE e CONIUGATE COINCIDENTI

$$Q(x) = a\left((x - x_1)^2 + y_1^2\right)^2$$

$$\frac{r(x)}{Q(x)} = \frac{A_1 x + A_2}{(x-x_1)^2 + y_1^2} + \frac{A_3 x + A_4}{\left((x-x_1)^2 + y_1^2\right)^2}$$

$$\int \frac{Ax+B}{(1+x^2)^2} \quad ; \quad \int \frac{x\,dx}{(1+x^2)^2} = \frac{1}{2}\int \frac{(1+x^2)'}{(1+x^2)^2}\,dx$$

$$= -\frac{1}{2}\,\frac{1}{(1+x^2)} + c$$

$$\int \frac{dx}{(1+x^2)^2} \quad ?$$

$$I_2(x) = \int \frac{1}{(1+x^2)^2}\, dx =$$

$$= \int \frac{(1+x^2)-x^2}{(1+x^2)^2}\, dx \quad =$$

$$= \int \left(\frac{1}{1+x^2} - \frac{x^2}{(1+x^2)^2} \right) dx =$$

$$= \int \frac{dx}{1+x^2} - \int \frac{\left(\frac{x^2}{2}\right)'}{(1+x^2)^2} \cdot x\,dx =$$

$$= \int \frac{dx}{1+x^2} - \frac{1}{2} \int \frac{(1+x^2)'}{(1+x^2)^2} \cdot x\,dx$$

$$= \operatorname{arctg}(x) - \frac{1}{2} \int \left(- \frac{1}{1+x^2} \right)' \cdot x\,dx$$

$$= \operatorname{arctg}(x) + \frac{1}{2} \int \left(\frac{1}{1+x^2} \right)' \cdot x\,dx$$

$$= \operatorname{arctg}(x) + \frac{1}{2} \frac{x}{1+x^2} - \frac{1}{2} \int \frac{1}{1+x^2} dx$$

$$= \frac{1}{2}\operatorname{arctg}(x) + \frac{1}{2}\frac{x}{1+x^2} + c$$

$$I_m(x) = \int \frac{dx}{(1+x^2)^m} =$$

$\boxed{m \geq 2}$

$$= \int \frac{1+x^2-x^2}{(1+x^2)^m}\,dx =$$

$$= \int \frac{dx}{(1+x^2)^{m-1}} - \int \frac{x^2}{(1+x^2)^m}\,dx =$$

$$= I_{m-1}(x) - \frac{1}{2}\int \frac{(x^2)'\,x}{(1+x^2)^m}\,dx =$$

$$= I_{m-1}(x) + \frac{1}{2(m-1)}\int \left(\frac{1}{(1+x^2)^{m-1}}\right)' x\,dx$$

$$= I_{n-1}(x) + \frac{1}{2(n-1)} \frac{x}{(1+x^2)^{n-1}} -$$

$$- \frac{1}{2(n-1)} \int \frac{dx}{(1+x^2)^{n-1}} =$$

$$= \left(1 - \frac{1}{2(n-1)}\right) I_{n-1}(x) + \frac{1}{2(n-1)} \frac{x}{(1+x^2)^{n-1}}$$

$$I_n(x) = \frac{2n-3}{2n-2} I_{n-1}(x) + \frac{1}{2(n-1)} \frac{x}{(1+x^2)^{n-1}}$$

$$\int \frac{dx}{(1+x^2)^3} = \frac{2\cdot 3-3}{2\cdot 3-2} I_2(x) + \frac{1}{2\cdot 2} \frac{x}{(1+x^2)^2} =$$

$$\frac{3}{4} I_2(x) + \frac{1}{4} \frac{x}{(1+x^2)^2}$$

$$\int \frac{x^2}{(1+x^3)^4}\,dx \underset{1+x^3=t}{=} \cdots = \frac{1}{3}\int \frac{dt}{t^4} = \cdots$$

$$\int \frac{x^3}{(1+x^2)^4}\,dx = \cdots \quad \text{Ex}$$

$t = 1+x^2$, $dt = 2x\,dx$

$x^3 = x^2 \cdot x$,

NOTA: $$\int \frac{1+2x+x^3}{(x-1)^4}\,dx \underset{\substack{x-1=t \\ dx=dt}}{=}$$

$$= \int \frac{1+2(t+1)+(t+1)^3}{t^4}\,dt = \cdots$$